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OPTIMAL SYSTEM SPARE CONFIGURATION BASED ON THE PRESENT WORTH OF OPERATIONAL COSTS UNDER A POLICY OF CANNIBALIZATION

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An economic model is developed for determining optimal spare provisioning requirements when a system is subject to cannibalization; in this context, cannibalization is defined as the process of using the good components of a terminally failed unit as a source of replenishment spares for future component failures in other units.

The model analyzes four components of a system's net operational costs: the manufacturing or procurement costs, the repair/replacement costs, the cannibalization costs, and a compensating revenue or return function, which is treated next page

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as a negative cost. The net cost is explicitly a function of the number of multi-component units deployed, the spare configuration for each component type, the time period over which the system is to be used, and the continuous annual interest rate. The model is present worth analysis, which reduces all future costs and revenue to a single equivalent present value. Consequently, alternative spare configurations must be compared over the same period of time.

The author develops a stochastic analysis of the repair/replacement and cannibalization processes, and derives general formulas for the time dependent repair/replacement, and cannibalization probability; the expressions are based upon general failure and replacement density functions. He also performs a State Transition Analysis, using an Absorbing State Markov Process, to model the consumption of spare components.

The cost model may be subject to one or more constraints, which provide bounds on the required number of spares for each component type.

An example is presented, and numerical solutions are obtained for the net present worth of the operational costs for several time horizons and interest rates. The respective present worth calculations are performed using the method of Gaussian Quadrature for the numerical integrations.

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CHAPTER I

INTRODUCTION

With the advent of complex equipment in both the government and industrial sectors of the economy, the study of system reliability and maintainability has become increasingly important. Complex systems, such as aircraft and mortar artillery locating radar systems, must frequently operate over long periods of time far from their sources of supply. It often happens that spares of a given part type are exhausted, while spares of another part type are in excess; subsequent failures of the part type with a deplete spare inventory will terminate system operation until replacements arrive from some external source of supply. This anomoly is a source of exhorbitant logistics support costs, when the cost of lost service or "downtime" is considered in the overall cost model of the system. For a system operating in the private industrial sector, the loss of system performance may alternatively be penalized with the dollar cost of lost revenue.

The potential cost of such lost service provides the motivation for developing a working policy for maintaining a particular system. The specific technique studied in this research is cannibalization.

The citations on the following pages follow the style of the Naval Research Logistics Quarterly.

Cannibalization, as defined in this context, is the replenishment of the spare inventory with working components from a terminally failed unit.

For purposes of illustration, consider a system made up of N independent and identical units, each composed of P serial components. When a component fails, it is replaced with a spare component from the inventory; if there are no available spares for this particular component type the unit is disassembled, the remaining working components are restored to original condition and added to their respective spare inventory locations. Thus, cannibalization is the procedure of disassembling the remaining working components, restoring, and adding them to the remaining spare component types. In effect, it is the dismantlement of units for components to be used as replacements in other units.

It must be realized, though, that this technique is appropriate and useful only when specific logistics problems, such as very large procurement lead times, prevent the acquisition of a replaceable component from the primary, or an alternate, source of supply. Otherwise it would obviously be appropriate to wait for the arrival of the the failed component type.

The traditional concept of cannibalization is one in which the working components of a failed unit are utilized directly as replacements for other units as their components successively fail. The primary difference between the "traditional" definition of cannibalization, and the one used here, is that in the traditional

definition the replacement activity must encompass both the disassembly from the donor system and subsequent installation into the failed system; whereas in the more restrictive definition used in this application, the replacement activity is defined strictly as a spare extraction from an inventory of discrete component types. The cannibalization activity is quantitatively defined in such a way to adequately describe the stochastic process of terminal component failure.

The traditional concept is not addressed in this research because it requires a more complex definition of the replacement time density function. Specifically, the replacement time of a terminally failed component within a working system would have to include the time required to locate and remove the correct part from the donor system as well as the time required for installation into the recipient system. The more restrictive definition of cannibalization gives a simpler stochastic definition of the probability of cannibalizing any unit because of the failure of a particular component type.

The technique of cannibalization can provide savings in the operational costs of a system because it limits or reduces the required number of initial spares for a working system; also, cannibalization offers a method for "re-using" most of the components of a terminally failed unit, which otherwise would have a small or negligible salvage value.

In today's competitive economic situation, doubly plagued

by the simultaneous effects of inflation and recession, it is not economically feasible to overdesign with respect to system reliability, maintainability, or availability. An optimal economic system design should yield a minimal cost while still meeting a minimum availability or reliability requirement. The system availability, $A(t)$, is defined as the probability that a system is functioning at any given point in time, t. On the other hand, the system reliability, $R(t)$, is the probability that a system has functioned successfully, that is, it has experienced no failures for an interval of time, $[0,t]$. Thus, in general, $R(t) \leq A(t)$.

Another feasible model constraint is a requirement that the probability of experiencing a critical spare shortage, which could require cannibalization of a particular unit, be strictly less than some preassigned value, say $1-\alpha$. This constraint can be stated alternatively as follows: the probability of no critical spare shortage for the system must be at least as large as α. By "critical spare shortage" is meant an inventory depletion which precludes further system operation upon subsequent failure of any component type.

It is the purpose of this research to quantitatively analyze cannibalization as a maintainability technique for minimizing the present worth of system operational costs over the system lifetime, subject to some constraint on system performance.

It is emphasized that the selection and application of the proper maintainability technique results in many economies measured

in terms of labor time, material, and money. These savings are attributable to the fact that maintainability prediction is considered to be a tool for design enhancement because it provides for the early recognition and elimination of areas of poor maintainability during the early stages of the design life cycle. Otherwise, areas of poor maintainability would only become apparent during demonstration testing or actual use, after which time correction of design deficiencies would be costly, and unduly delay schedules and missions.

Besides simply being a maintainability technique, this formulation of cannibalization may be implemented as a design tool for the initial system spare configuration which would minimize the overall operating costs to the user. It is especially appropriate to systems which experience deterioration and obsolescence with time as was evidenced with the B-52 and DC-3 aircraft. A functional policy of cannibalization may have been able to prolong the useful life of such aircraft in Southeast Asia with a smaller expense to the Defense Department.

Research Objectives and Assumptions

The primary goal of this research is to develop an operational cost model which allows one to consider cannibalization as a maintainability technique. The analysis, based on specific component failure and replacement density functions, is a systematic presentation of the cannibalization technique, and its consequences in determining the optimal spare inventory allocation for a system of N independent and identical units, each with P discrete components or modules per unit. The optimal spare inventory allocation is the one with the minimal present worth of the net operational costs, yet satisfies a specific constraint on overall performance, such as meeting a minimum level of overall availability, or a requirement that the probability of no critical spare shortage for the system be at least as large as a pre-assigned value, say α_0, specified by the system user. Because the number of spares are finite, a constraint on system availability is not mathematically feasible, except in the steady state when the availability reaches zero for a finite number of units and spares.

A possible mathematical formulation of the problem is stated as follows:

$$\text{Minimize } K_0 = K_1 + K_2 + K_3 - \int_0^T \sum_{i=0}^{N} V_i(t)\, P_i(t) \exp(-rt)\, dt,$$

where K_0 is the present worth of the system operational costs, K_1 is the present worth of the manufacturing or procurement costs, K_2 is

the present worth of the replacement costs, and K_3 is the present worth of the cannibalization costs. $V_i(t)$ is the system return or revenue function when i units are operating at time t, $P_i(t)$ is the probability of exactly i units operating at time t.

The present worth of the return or revenue function is given by the integral of the product of the expected return for having at least 0 units operating, with the continuous compounding discount factor, exp(-rt). The present worth calculation is performed over the closed interval $[0,T]$, where T is the specified system time horizon.

The development of the model will be subject to the following assumptions:

1. Each of the N units is identical, and consists of P components.
2. The system is supported with an inventory of spares, with m_j spares initially stocked for the j^{th} component type, $1 \leq j \leq P$. Spares are not subject to failure.
3. All failures are independent and the time to failure for the j^{th} component type is governed by the exponential probability density function, $f_j(t) = \lambda_j \exp(-\lambda_j t)$, for $1 \leq j \leq P$.
4. Failures are detected instantly and replacement commences immediately if a spare for the particular component type is available, otherwise the unit is cannibalized.

5. The replacement times are independently distributed and governed by the probability density function $g_j(t)$, for each of the P component types, $1 \leq j \leq P$; the average replacement time is much less than the mean time between failures.

6. Each component type has its own repair facility capable of performing repairs upon demand; there are no queueing effects.

7. The statistical parameters of the failure and replacement density functions are known.

8. When a unit is cannibalized, the (P-1) remaining operable components are refurbished and become part of the homogeneous spare inventory, indistinguishable from the unused spares.

9. Component salvage values are negligible.

10. System operational costs consist only of manufacturing or procurement costs, replacement costs, cannibalization costs, and a revenue or return function which is treated as a negative cost.

Dissertation Outline

Chapter II contains a thorough review of the currently published literature on cost modelling and maintainability analysis.

Chapter III develops the overall system cost model for a general system configuration and contains a stochastic description of the process of cannibalization. It results in an explicit equation for the cannibalization probability as a function of time. Also included in Chapter III are some example cost calculations and a discussion of possible revenue or return functions.

Chapter IV outlines applicable system constraints and illustrates, in detail, the required calculations for a constraint on spare adequacy. Other constraints discussed are limitations on the total weight or volume of a specific spare allocation, a resource constraint on the total investment in spares, a requirement on a system's interval availability, and a maintainability constraint on system downtime.

Chapter V contains the necessary criteria for applying this cost model and analyzes an example problem in detail. Also included in Chapter V is the State Transition Analysis necessary to calculate $P_i(t)$, the probability of exactly i units operating at time t.

Chapter VI contains a concluding discussion, and suggestions for further research.

CHAPTER II

REVIEW OF THE LITERATURE

The following review represents a thorough search of the currently published works in the fields of reliability and maintainability which are related to the use of system cannibalization as a maintainability technique. Since very little research has been published dealing directly with cannibalization as it is currently defined, the literature review addresses those articles which consider stochastic system analysis from both a cost and performance viewpoint.

A rather detailed analysis of the effects of cannibalization on the optimal transition of multi-unit-component systems was done by Brown[7] . The optimal transition policy was the one which yielded the maximum net revenue to the system user over a prescribed system lifetime. The policy determined if and when to retire a system from active use, and use it as a source of spares for other fielded systems. In this work, he developed an algorithm based on the present worth of the system return as the decision criterion. He did not consider the overall system availability or the probability of spare adequacy as constraints in his model.

Brown made nine general assumptions in his model:

1. The spares are not subject to failure.

2. All the components of a unit must be functioning for a unit to be available.
3. The terminal failures of the components are statistically independent.
4. The replacement or cannibalization of a component does not cause failures.
5. Replacement times for failed components are sufficiently short to not affect the availability of a unit. (This is equivalent to an instantaneous replacement model.)
6. The parameters required by the model can be determined.
7. Only random failures of the components are considered.
8. Failures are amenable to the Poisson postulates thus having an exponential distribution of time to failure.
9. Salvage values are negligible.

Although Brown acknowledged that there existed a time dependent probability of cannibalization, as well as a time dependent probability of replacement, he never developed a closed form description of these functions.

Brown also considered the incremental changes in the system return function due to incremental changes in the number of spares for any component type. He also discussed the effects of perturbing the number of units on the system return function. Both analyses were done using mathematical induction rather than any generalized closed form analysis or system simulation.

Gaver [15] studied the availability of some simple redundant systems, in which repair was possible in case of failure. The systems discussed were composed of two identical subsystems, and the system was considered to be in a state of failure when both subsystems were simultaneously in such a state. Gaver's work was significant in that it showed the relative gains possible in the mean time to system failure when repair facilities are added.

Wilken and Langford [51] determined the probability of exactly i failures occurring in a time period of length t, P(i,m,N,t), for a system of N identical single component units in parallel with m spares. They derived the following equation, which is correct for exponential failures and instantaneous replacement.

$$P(i,m,N,t) = \begin{cases} (N\lambda t)^{i} \exp(-N\lambda t)/i!, & \text{for } 0 \leq i \leq m. \\ \dfrac{N^{m}\ N!}{(i-m)!(N-i+m)!} \displaystyle\sum_{j=0}^{i-m-1} \dfrac{(-1)^{j}\ (i-m)!}{j!(j-i+m)!} \times \\ \qquad \left[\dfrac{\exp((i-m-j)\lambda t)}{(i-m-j)^{m}} - \displaystyle\sum_{k=0}^{m} (i-m-j)^{k-m} \dfrac{(\lambda t)^{k}}{k!} \right] & \text{for } i+m \leq i \leq m+N. \\ 0 & \text{for } i > m+N \end{cases}$$

Guillory [18] illustrated a policy of backrobbing, in which a unit or sub-system is removed from the repair queue in order to expedite the repair of a failed system inside the service facility. The technique was used when the inventory of spares was deficient and the repair time would cause delay in the repair of the system.

Guillory's analysis was important primarily from the standpoint of expediting the repair of critical systems. It emphasized the stochastic behavior of systems within a repair queue; however it did not fully address the aspect of operational costs and spares provisioning, as is intended by this research.

Hirsch, Meisner and Boll [19] developed a "structure function" in order to obtain an effective method for determing the probability density function of the performance levels of a system subject to cannibalization. They assumed that parts either "work" or "fail", but allowed the possibility that the system may function at any one of several levels of performance.

They studied the probability distribution of system states in the two special cases:

1. No spares are available.
2. Spare part inventories are arbitrary, and failures of parts are governed by the Poisson Process.

They also made the following two assumptions:

1. Failures are detected instantaneously, and part replacements and interchanges are performed instantaneously.
2. Part lifetimes are independent random variables, and parts of the same type have identical lifetime distributions which are not affected by cannibalization.

Again, the work of Hirsch, Meisner, and Boll was primarily one of developing a particular stochastic description of the process of cannibalization without regard to the potential savings, in terms of decreased system operational costs throughout the life of the system.

Simon [40, 41] generalized the model of Hirsch, Meisner and Boll and developed an "admissible" cannibalization policy which allocated shortages to maximize the system state following each failure. He showed that not all admissible policies are equally desirable.

Simon's work was essentially an extension of the model described by Hirsch, Meisner, and Boll, with no emphasis on the aspects of overall system costs or the provisioning of spares.

Black and Proschan [6] have considered the problem of assigning spares in some optimal fashion to several subsystems of dissimilar components that constitute a larger system. They allocate some total number of available spares in such a way that system reliability, or the probability that final system failure will not occur prior to some specified time, is at a maximum. However, their optimal solution was with respect to system reliability, not overall system operational costs.

McNichols and Wortham [24] presented a decision making model based upon the present worth of selected income patterns in order to provide a method for evaluating projects with alternative income patterns over time. They also provided a technique which showed the possibility of attaining various levels of income for each particular time duration specified by the project manager. Their results were based upon assumed income patterns; specifically, generalized increasing income rates, generalized decreasing income rates, geometric, and arithmetic income growth rates. Their present worth analysis could be applied to cost calculations by considering a particular cost pattern as a negative income pattern; consequently, the present worth of this revised income pattern would be negative, implying an expenditure rather than a net inflow of funds.

Similarly, the research results presented in this Dissertation are based upon the present worth of particular operational costs as the decision criterion for the system configuration. However, no generalized operational cost patterns are assumed; rather, an expression for the present worth of the system operational costs is derived. In the analysis presented herein, the assumed revenue function is treated as a negative cost.

McNichols and Messer [23] developed a procedure for allocating "availability parameters (repair times and failure rates)" to the specific components of series systems in order to minimize the cost of system development. They used the Lagrange Multiplier technique of constrained optimization assuming constant failure rates and repair times. Their cost function specifically delineated the cost incurred in order to decrease the failure rates of each of the components as well as the cost of decreasing the repair times. The authors labeled these costs "component improvement costs".

They solved for the optimal system parameters of failure rate and repair time for each of the respective components which minimized the system's improvement cost required to meet a particular availability level. They also investigated the effects of holding a parameter (failure rate) at a fixed value while allowing the parameters of the remaining components to change. They pointed out that the constant failure rate assumption was necessary in order to develop and optimize the analytic model.

They did not investigate, however, the effects of sparing and

downtime upon the overall system's cost, because the model was to be implemented during an equipment's design stage when knowledge of sparing and downtime costs is limited.

Morrison [29] considered a complex system whose components were divided into two subsystems, each containing components whose lives were exponentially distributed with different scale parameters. The components were assumed connected in series, thus failure of any component caused the entire system to fail. Each of the subsystems was unique and supplied with an inventory of spares. System failures were corrected by replacing failed subsystem components with components from the appropriate spare inventory. When a failure occurred with the corresponding spare inventory deplete, the entire system was discarded. No form of cannibalization was considered because there were only two unique subsystems, each with different failure density parameters, and presumably, different functional characteristics. He gave an explicit solution to the specific problem of allocating spares to maximize the expected life of a system with an inventory of similar spares. His analysis was restricted to components with an exponential failure density. He developed tables of the maximum expected system life under various spare allocations for each of the component types. He also allocated spares for maximum reliability.

Again, this work was primarily directed to allocating spares to maximum reliability without considering system costs. It also

neglects the density function of time to repair.

Morrison and David [30] obtained the distribution of operating life for a series system of like elements supplied with a set of spare components. This distribution has been evaluated for some common types of component life densities, exponential and gamma, and tables of expected total system life have been constructed. These expectations were compared with those of systems with no spares as a measure of the efficacy of the additional spare components. They also studied the reliability of systems with spares.

The report by Morrison and David is similar to the research of this Dissertation, in that several failure distributions were considered, and the system configuration was initially a series system of identical elements, supplied with a set of spare components. The system efficacy was measured by comparing the expected system life for a system with a particular number of spares to the expected life of a system with no spares. However, as in most of the research cited so far, no emphasis was placed on the cost of deploying and operating a particular system.

Crow [12] considered the theoretical and practical implications of the nonhomogeneous Poisson Process model for reliability, and gave estimation, hypothesis testing, comparison and goodness of fit procedures when the process had a Weibull probability density function. Applications of the Weibull model in the field of reliability and in other areas were discussed.

Crow postulated that, for a repairable system, one is rarely

interested primarily in the time to first failure. Rather, he emphasized the interest in the probability of system failure as a function of system age. He pointed out that often mathematical idealizations assume that the failure times of the complex repairable system follow a Poisson Process.

For various repairable systems, particularly of the complex electronic type, a constant failure rate had been considered representative, after perhaps an initial burn in period. The period in which a system exhibits a constant intensity of failure is often called the system "useful life". If a complex system consists of a large number of components, each acting independently, if the failure of a component results in a failure of the system, and if each component is replaced upon failure, then under fairly general conditions, the occurrence of failures of the system will approach a constant intensity, or rate, as the number of components and operating time become large. However, many large or complex repairable systems generally experience a wearout phase which eventually makes them economically impractical or too unreliable to continue in service without undergoing overhaul. Such systems, according to Crow, will never achieve the equilibrium state of a homogeneous Poisson Process.

Crow's model was primarily directed to a stochastic system description, rather than to a cost modelling methodology as is a primary objective of this research.

Timsans et al. [45] developed a family of hyperbolic cost functions which could be used to analyze the cost of achieving prescribed levels of reliability and maintainability, for series systems experiencing constant failure rates. The required failure rates, and mean repair times, were determined in order to minimize the total system cost yet meet an availability constraint. They used a Lagrange Multiplier technique essentially equivalent to that used by McNichols and Messer [23] to solve the allocation problem.

Cleroux and Hanscom [9] analyzed age replacement policies where replacement of a unit occurs at failure, or at age T, whichever occurs first. The age replacement policy which minimized the average expected cost per unit time over an infinite time span was obtained in the case where the cost structure involved a term accounting for adjustment costs or interest charges which were incurred at fixed intervals of time of equal length. The adjustment costs included maintenance expenses required to keep the equipment operating, and depreciation costs resulting from the decline in resale value due to age and condition. However, no consideration was given to the density of time to perform repairs and replacements.

Proschan [33] solved the inventory problem of maximizing spare adequacy subject to a single linear constraint on the total investment in spares. He modelled the process with exponential failures and instantaneous replenishment.

He also gave a thorough discussion of the use of Polya type distributions in Renewal Theory. He proved that the Polya distribution family is closed, i.e. the convolution of a Polya type m function with a Polya type n function is a Polya function of type: Min(m,n).

None of the previous works, which dealt with the stochastic analysis of systems subject to failure and repair, included the repair/replacement density function in the analysis. Instantaneous replenishment, or fixed repair times, were generally assumed in most of the stochastic models developed. Consequently, a cost model formulated using the relevant repair/replacement density functions is a significant contribution to the state of the art, and extends the scope of the current literature. The analysis contained herein is such a contribution.

The author develops a stochastic analysis of the repair/replacement and cannibalization processes, and derives general formulas for the time dependent repair/replacement, and cannibalization probability; the expressions are based upon general failure and replacement density functions. He also performs a State Transition Analysis, using an Absorbing State Markov Process, to model the consumption of spare components.

The cost model may be subject to one or more constraints, which provide bounds on the required number of spares for each component type.

An example is presented, and numerical solutions are obtained for the net present worth of the operational costs for several time horizons and interest rates. The respective present worth calculations are performed using the method of Gaussian Quadrature for the numerical integrations.

CHAPTER III

SYSTEM CONFIGURATION AND OVERALL COST MODEL

System Configuration

The system configuration is depicted in Figure 3.1. The system consists of N identical units, each with P component types. Each of the P component types is supported with an inventory of m_j spares, $1 \leq j \leq P$.

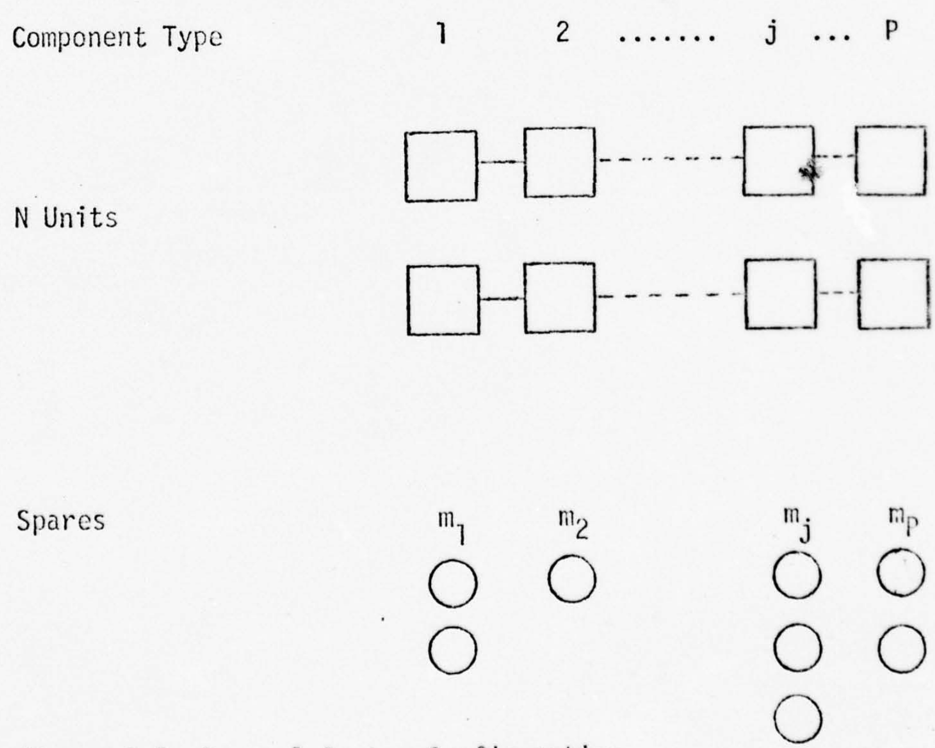


Figure 3.1 General System Configuration

The schematic illustration of Figure 3.1 could represent a fleet of aircraft, cars, or ships, in which the component types represent basic subdivisions of the entire vehicle, or unit. However, the schematic is a more practical and realistic depiction of a group of identical radio receivers, each composed of P "plug-in" modules. Obviously, any particular class of radios has a fixed number of modules required for proper operation. This fixed number of required modules is P.

For a system of radios, for example, it is reasonable for each of the P major modules, or components as they will now be called, to be supported by an inventory of spares. There are initially m_j spares for the j^{th} component type, $1 \leq j \leq P$. Qualitatively, it would be assumed that those components with the largest failure rates would be stocked with the largest number of spares; however, this may not be necessarily true, since system availability, or the probability that a specified number of units is operational at a specific time, is not the major criterion for determining the best system configuration. The present worth of the system's operational costs is the major criterion of system worth.

As was specified in the introduction, the cost model for study in this research is comprised of the sum of the present worths of the manufacturing or procurement costs, the repair or replacement costs, and the cannibalization costs; the system revenue or return function is considered as a negative cost to offset the other costs.

Manufacturing Costs

The manufacturing, or procurement, costs are assumed to be a linear function of the number of components of each type. Thus, if a_j is the unit cost of producing or procuring the j^{th} component type, then the cost incurred at time zero is:

$$(3.1) \quad K_1 = N \sum_{j=1}^{P} a_j + \sum_{j=1}^{P} a_j m_j \ .$$

K_1 is the present worth of the manufacturing costs incurred in order to begin system operation. The first term represents the cost incurred to obtain the working system of N independent and identical units, while the second expression is the cost of the spares for all of the P component types.

The expression given previously for the initial manufacturing costs, Equation (3.1), neglects such factors as quantity discounts, in which the component costs of the j^{th} component type, a_j, decreases for large quantities procured. It also neglects any production "learning curve" effects in which the component production cost decreases with increases in the production yield; the increased production yield is partially a result of improved manufacturing techniques.

In applying this model for the manufacturing costs it is possible to calculate in such a way that it incorporates all the fixed and variable costs of manufacturing, as well as the trans-

portation, set-up, and any other costs incurred upon initially deploying the system.

On a present worth basis, the manufacturing costs will be valued at their initial level because they are incurred only at time zero. It is assumed that no additional spares are manufactured or procured after the initial system is set up. Thus, the present worth of the manufacturing costs does not depend upon the nominal annual interest rate, r, nor the time horizon, T. Equation (3.1) is the expression for the present worth of the initial cost of manufacturing or procuring the system depicted in Figure 3.1.

Replacement Costs

The replacement costs are the costs incurred with replacing failed components with components from the respective component spare inventory. Each component type, j, will have associated with it a replacement cost, $b_j(t)$, which, in general, may be a function of time. The coefficient $b_j(t)$ is the component replacement cost for a single replacement of the j^{th} component type. It is primarily a personnel or labor intensive cost, since its greatest portion is assumed to be composed of the wages and incidental personnel costs of those performing the replacement.

The replacement cost is defined as the cost incurred in locating the failed component within the failed unit, removing it, and installing a new spare. The removed component is not repaired, but discarded; it is assumed to have zero salvage value in the

failed state.

The total component replacement cost is a function of both the cost to replace a particular component type, and the probability that the specific component has been repaired. In the context of this analysis, the terms "repair" and "replacement" will be used synonymously.

In the case of a finite repair time governed by the probability density function $g_j(t)$, the total expected cost of repair for the system configuration of Figure 3.1 is:

$$(3.2)\quad E(\text{Repair Cost}(t)) = \sum_{j=1}^{P} b_j(t) \sum_{i=1}^{N+m_j-1} i \; Pr\left\{\begin{array}{l} i \text{ repairs of the} \\ j^{th} \text{ component type} \\ \text{by time } t \end{array}\right\}$$

The coefficient $b_j(t)$ is the cost of a single replacement of the j^{th} component type.

Figure 3.2 contains the chronological description of one failure-repair cycle occurring before time t. The probability of such an occurrence for the j^{th} component type is calculated as follows:

Figure 3.2 Chronological Description of Exactly 1 Failure-Repair Completion by Time t.

$$(3.3)\quad \Pr\left\{\begin{array}{c}\text{Failure between}\\ \text{x and x+dx, and repair}\\ \text{between y and y+dy, and}\\ \text{no failures between}\\ \text{y and t.}\end{array}\right\} = f_j^{(1)}(x)dx\,g_j(y-x)dy\,R_j^{(1)}(t-y).$$

Integrating over all $0 \leq x \leq y$, and $0 \leq y \leq t$, one obtains

$$(3.4)\quad \Pr\left\{\begin{array}{c}\text{1 failure}\\ \text{and}\\ \text{1 repair}\end{array}\right\} = \int_0^t \int_0^y f_j^{(1)}(x)\; g_j(y-x)\; R_j^{(1)}(t-y)\; dxdy$$

$$= f_j^{(1)}(t) * g_j(t) * R_j^{(1)}(t), \text{ where}$$

$f_j^{(1)}(t)$ is the probability density function of the time to the first failure, and $R_j^{(1)}(t)$ is the effective reliability function associated with $f_j^{(1)}(t)$.[1] The replacement probability is a multiple convolution, where the convolution operation is defined as follows:

$$(3.5)\quad A(t) * B(t) = \int_0^t A(x)\; B(t-x)\; dx,\; t \geq 0.$$

In general, convolution is commutative, distributive, and associative as illustrated by the following equations, which are presented for reference purposes without proof:

$$(3.6)\quad A(t) * B(t) = B(t) * A(t)\,.$$

$$(3.7)\quad A(t) * (B(t) + C(t)) = A(t) * B(t) + A(t) * C(t)\,.$$

$$(3.8)\quad A(t) * (B(t) * C(t)) = (A(t) * B(t)) * C(t)\,.$$

[1] Appendix I contains the derivation of $f_j^{(1)}(t)$ and $R_j^{(1)}(t)$ as used in this analysis.

Continuing, the probability of exactly 2 failures and 2 repairs for the j^{th} component type, by time t, is:

$$(3.9) \quad Pr\left\{\begin{matrix}\text{2 failures}\\ \text{and}\\ \text{2 repairs}\end{matrix}\right\} = f_j^{(1)}(t)*g_j(t)*f_j^{(1)}(t)*g_j(t)*R_j^{(1)}(t)$$
$$= (f_j^{(1)}(t)*g_j(t))^{*2} * R_j^{(1)}(t).$$

Figure 3.3 contains the description of all possible failure and repair modes for the j^{th} component type. Allowing cannibalization, there can be as many as $N+m_j-1$ failure-repair cycles for the j^{th} component type, versus m_j replacements without cannibalization.

In general, the probability of exactly i failures, i repair completions, with the component being operational by time t is:

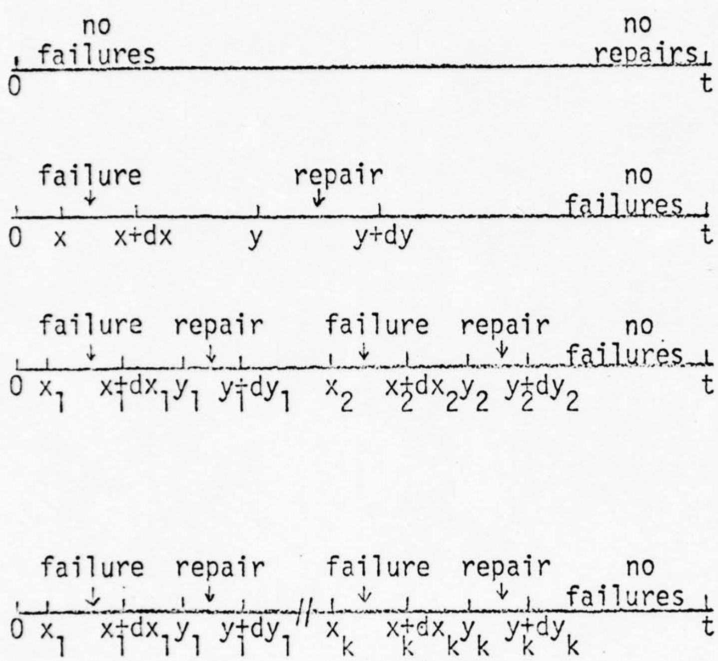
$$(3.10) \quad Pr\,\{\text{i failures, i repairs}\} = (f_j^{(1)}(t)*g_j(t))^{*i} * R_j^{(1)}(t),$$

where $f_j^{(1)}(t)$ and $g_j(t)$ are the component's ordered failure and repair density functions, respectively; $R_j^{(1)}(t)$ is the "ordered" reliability function of the j^{th} component type. The superscript, $*i$, indicates the i^{th} multiple convolution.

Substituting the expression for the probability of i failures and i repair completions into Equation (3.2) yields a closed form expression for the expected repair cost as a function of time.

$$(3.11) \quad E(\text{Repair Cost}(t)) = \sum_{j=1}^{P} b_j(t) \sum_{i=1}^{N+m_j-1} i(f_j(t) * g_j(t))^{*i} * R_j(t).$$

In general, the present worth of a system's repair/replacement cost is obtained by integrating the product of the expected repair



Number of Repairs by Time t	Stochastic Description	Probability of Repair Completion
0		0
1		$f_j^{(1)}(t)*g_j(t)*R_j^{(1)}(t)$
2		$(f_j^{(1)}(t)*g_j(t))^{*2}*R_j^{(1)}(t)$
.		
.		
.		
$N+m_j-1$		$(f_j^{(1)}(t)*g_j(t))^{*k}*R_j^{(1)}(t)$ where $k = N+m_j-1$.

Figure 3.3 Description of Possible Failure-Repair Cycles for the j^{th} Component Type.

cost, with the term exp(-rt) over the required time horizon, $[0,T]$. The term exp(-rt) is the continuous annual compounding discount factor, where r is the nominal annual interest rate.

The present worth of the system's repair/replacement costs is designated K_2.

(3.12) K_2 =

$$\int_0^T \sum_{j=1}^{P} b_j(t) \sum_{i=1}^{N+m_j-1} \left[\{ i(f_j^{(1)}(t)*g_j(t))^{*i} \} * R_j^{(1)}(t)\right] \exp(-rt)\, dt.$$

Cannibalization Costs

The total expected cost of cannibalization for a system composed of N units, depends upon the probability of cannibalizing a unit as well as the cannibalization cost incurred when a particular component fails causing cannibalization. The cannibalization probability is a function of the number of units, N, and the number of spares, m_j, for each component type, j.

The total expected cannibalization cost is expressed as a sum of products of the component cannibalization cost, c_j, and the probability of that particular component failing and causing cannibalization of unit i, $D_{ij}(t)$. There can be as many as (N-1) cannibalizations of any system initially composed of N units.

(3.13) $$E(\text{Cannibalization Cost } (t)) = \sum_{i=1}^{N-1} \sum_{j=1}^{P} c_j D_{ij}(t).$$

Since each unit is assumed identical, and the periodic replacement of failed components does not significantly alter the configuration of any unit, then the cannibalization probability is the same for any unit within the system. It only varies with the component type, j, $1 \leq j \leq P$. Thus $D_{ij}(t) = D_j(t)$. The cannibalization probability, $D_j(t)$ will be expressed using the density functions of the ordered times to failure of the j^{th} component type.

The revised cannibalization cost equation is:

$$(3.14) \quad E(\text{Cannibalization Cost}(t)) = (N-1) \sum_{j=1}^{P} c_j D_j(t)$$

This revised equation results from the basic premise that as many as (N-1) units may be cannibalized, each with equal liklihood.

Stochastic Description of Cannibalization

The probability expression, $D_j(t)$, will be expressed using the marginal density functions of the ordered failure times of the j^{th} component type, $1 \leq j \leq P$. It can be shown, however, that for a very large time horizon, the probability of cannibalizing a unit because of failure of a particular component type is not a function of time, but depends upon the number of units, N, the number of spares, m_j, and the respective hazard rate for each component type j, $1 \leq j \leq P$.

For a system initially composed of N units, with P components per unit, and m_j spares for the j^{th} component type, $1 \leq j \leq P$, a unit is cannibalized if $(N+m_j-1)$ failures occur in component type j

before $(N+m_k-1)$ failures occur in any other component type, k, with $k \neq j$, $1 \leq k \leq P$. Thus, $D_j(t)$ is the probability that the $(N+m_j-1)^{th}$ failure time of the j^{th} component type is less than the smallest of the $(N+m_k-1)$ failure times of the remaining component types; then it is, by definition, less than all of the $(N+m_k-1)$ failure times of the remaining (P-1) components.

The following analysis is based upon independent identically distributed times to failure for the j^{th} component type. Define $t_j^{(N+m_j-1)}$ as the time to the $(N+m_j-1)^{th}$ failure of the j^{th} component type, and $D_j(t)$ as the probability that the j^{th} component type caused the cannibalization.

$$(3.15) \qquad D_j(t) = Pr\{ t_j^{(N+m_j-1)} < \min_{\substack{k=1,..,P \\ k \neq j}} (t_k^{(N+m_k-1)}, .,.,.)\}$$

If the $(N+m_j-1)^{th}$ failure of the j^{th} component type is less than the minimum of the corresponding failure times of the remaining components, then it is less than all of the corresponding failure times of the remaining components.

$$(3.16) \qquad D_j(t) = Pr\{ t_j^{(N+m_j-1)} < \mathrm{All}_{\substack{k=1,..,P \\ k \neq j}} (t_k^{(N+m_k-1)}, .,.,.,)\}$$

Under the assumption of independent component failures, the probability that the $(N+m_j-1)^{th}$ failure time of the j^{th} component type is less than all of the corresponding failure times of the remaining component types is the product of the probabilities

that the $(N+m_j-1)^{th}$ failure time of the j^{th} component type is less than each of the $(N+m_k-1)$ failure times of the k component types, for $k = 1, 2, \ldots, P, k \neq j$.

$$(3.17) \quad D_j(t) = \prod_{\substack{k=1 \\ k\neq j}}^{P} \Pr \{t_j^{(N+m_j-1)} < t_k^{(N+m_k-1)}\} \quad .$$

The probability expression given in Equation (3.17) is calculated using the marginal density function of the time to the $(N+m_j-1)^{th}$ failure of the j^{th} component type. The required marginal density function of the ordered failure times is obtained by integrating the joint density function of the respective component ordered failure times; the joint density function is $h(t_1^{(N+m_1-1)}, t_2^{(N+m_2-1)}, \ldots, t_p^{(N+m_p-1)})$. Because of the independence of the respective ordered failure times, the joint density function of the ordered times to failure is the product of the respective marginal density functions, $h_1^{(N+m_1-1)}(t)$, $h_2^{(N+m_2-1)}(t), \ldots,$ etc.

Since the marginal density functions of the ordered times to failure of each of the component types are proper density functions, the definite integrals of the marginal density functions, $h_1^{(N+m_1-1)}(t)$, $h_2^{(N+m_2-1)}(t)$, etc., from zero to infinity, all reduce to unity. Consequently, the following double integration results:

$$(3.18) \quad \Pr\{ t_j^{(N+m_j-1)} < t_k^{(N+m_k-1)} \} = \int_0^t \int_0^y h_j^{(N+m_j-1)}(x) \; h_k^{(N+m_k-1)}(y) \; dx \; dy.$$

Using previous reasoning, the following expression yields the time dependent probability of a cannibalization due to a failure of the j^{th} component type.

$$(3.19) \quad D_j(t) = \prod_{\substack{k=1 \\ k \neq j}}^{P} \int_0^t \int_0^y h_j^{(N+m_j-1)}(x) \; h_k^{(N+m_k-1)}(y) \; dxdy \quad .$$

If the mean times to repair a component type are significantly less than the respective mean time between failures, it is possible to make use of an instantaneous replenishment model, in which the marginal density functions, $h_j^{(i)}(t)$, for $j = 1, 2, ..., P$, and $i = 1, 2, ...N+m_j-1..$, are equivalent to the probability density functions of the i^{th} ordered failure of the j^{th} component type, i.e. $f_j^{(i)}(t)$.

Case of Finite Replacement Time

In the case of a finite replacement time governed by the density function $g_j(t)$, the joint density function of the ordered times to the $(N+m_j-1)^{th}$ failure must include the time to complete the replacement of each failed component type. The marginal density function

of the time to the i^{th} failure of the j^{th} component type, $h_j^{(i)}(t)$, is the density function of the sum of the times of the i ordered failures, and the (i-1) repairs of the j^{th} component type.

Under the condition of a non-zero replacement time, or finite replacement rate, with replacement time governed by the density function $g_j(t)$, it is not necessarily true that $h_j^{(i)}(t) = f_j^{(i)}(t)$. The density function of the sum of the i independent failure times and (i-1) independent repair times is the convolution of the i-fold convolution of the ordered failure density with the (i-1)-fold convolution of the repair density.

$$(3.20) \qquad h_j^{(i)}(t) = f_j^{(1)}(t)^{*i} * g_j(t)^{*(i-1)}, \quad 1 \le j \le P, \quad 1 \le i \le N+m_j-1.$$

For example, the time to the third failure of the first component type will be the sum of the first three failure times and the first two repair times of the first component type. In this case, the marginal density function of the time to the third failure of the first component type is expressed as a multiple convolution of the ordered failure and repair density functions.

$$(3.21) \qquad h_1^{(3)}(t) = f_1^{(1)}(t)*g_1(t)*f_1^{(1)}(t)*g_1(t)*f_1^{(1)}(t)$$

$$= f_1^{(1)}(t)^{*3} * g_1(t)^{*2}.$$

In terms of the marginal densities of the terminal failure

times, $h_j^{(N+m_j-1)}(t)$ and $h_k^{(N+m_k-1)}(t)$, the cannibalization probability, $D_j(t)$, is expressed as follows:

$$(3.22) \quad D_j(t) = \prod_{\substack{k=1 \\ k \neq j}}^{P} \int_0^t \int_0^y h_j^{(N+m_j-1)}(x)\, h_k^{(N+m_k-1)}(y)\, dxdy \quad .$$

Substituting the correct form of the convolved failure and repair density functions into the marginal density functions yields the correct expression for the cannibalization probability function:

$$(3.23) \quad D_j(t) = \prod_{\substack{k=1 \\ k \neq j}}^{P} \int_0^t \int_0^y \left(f_j^{(1)}(x)^{*(N+m_j-1)} * g_j(x)^{*(N+m_j-2)}\right) X$$

$$\left(f_k^{(1)}(y)^{*(N+m_k-1)} * g_k(y)^{*(N+m_k-2)}\right) dx\, dy.$$

Present Worth of Cannibalization Cost

Define K_3 as the present worth of the cannibalization cost. Assume c_j is the cost of cannibalizing a unit due to failure of the j^{th} component type, which occurs with probability $D_j(t)$. Also assume that the nominal annual interest rate is r, and that interest is compounded continuously.

Based upon Equation (3.14), the present worth of the expected cannibalization costs, over a time horizon T, is expressed as follows:

$$(3.24) \quad K_3 = \int_0^T (N-1) \sum_{j=1}^{P} c_j D_j(t) \exp(-rt)\, dt.$$

K_3 is explicitly a function of the system size, as measured by the number of units, N, and the number of components per unit, P. The cost is implicitly a function of the spare inventory size, m_j, since the term $D_j(t)$ is a function of both N, and m_j.

The coefficient c_j is determined empirically from previous cannibalization activity on a given system. The time horizon is estimated by the system user, and will be based upon previous data on similar systems. It could also be determined by the system designer before the specification requirements for the system are written.

System Revenue or Return Functions

The basis for calculating a system's return or worth to the user is the present worth of the instantaneous revenue function. The system revenue, or return function $V_i(t)$, is a monetary measure of the value of i units to the system user when they are operational at time t. If it is not measured in "dollars per unit time", for example a production process whose output is measured in "output per unit time", then it should be converted to a monetary measure before using this model.

Brown[7] justified a monotonically decreasing revenue function

with increasing age, on the basis that increasing obsolescence and decreasing efficiency will usually cause a monotonic decrease in the revenue function. Also with the attrition of units, the revenue function of the system usually decreases, and continued depletion of the system could result in a system having too few units to operate successfully. This is exemplified by a Taxi service having too many vehicles undergoing repair; the result would be long waiting times for current customers who may utilmately switch to another Taxi system, resulting in decreased revenue for the original Taxi service.

Even though the system's instantaneous revenue function, $V_i(t)$, will usually increase monotonically for any increase in the number of operating units, and the revenue function could increase proportionately for initial increases in the number of units, additional units will eventually be of less value.

The law of diminishing marginal returns states that as equal increments of one input are added to a production system, with the inputs to the other productive services (remaining units) being held constant, the resulting incremental production (system revenue) will decrease. However, the law of diminishing marginal returns says nothing about the effects of increasing the number of units whenever technological processes, which could alter the revenue producing capability of subsequent units, are changing. The units are assumed to be operating in a stable technological environment throughout the time period of interest, so the law of diminishing

marginal returns is applicable.

A consequence of the law of diminishing marginal returns is to partially limit, or provide an upper bound to, the number of units initially deployed, N; the upper bound is the number of units for which the marginal operating cost of the system just equals the marginal revenue derived from system operation. This microeconomic result would fix the optimal number of units initially deployed to a point where the marginal revenue obtained from operating the N^{th} unit was equal to the marginal cost of operating that unit. The marginal cost would be the incremental manufacturing costs, plus the expected repair costs for the last unit, and the expected cannibalization costs for the last unit deployed.

However, this microeconomic analysis does not account for the stochastic system behavior. Specifically, it assumes that the availability of each of the units is unity; the system would be assumed to operate deterministically, with N units available throughout its life.

Since the unit availability function $P_i(t)$, is the probability of exactly i units being operational at time t, the revenue obtained from having i units operational is simply the product of the unit revenue function, $V_i(t)$, and the unit availability function, $P_i(t)$, i.e. $V_i(t)\ P_i(t)$.

The unit revenue function, $V_i(t)$, is said to be a monotonically increasing function of the number of units, i, if for any two values,

i_1 and i_2, with $i_1 < i_2$, the relation $V_{i_1}(t) \leq V_{i_2}(t)$ holds. $V_i(t)$ would be strictly monotonically increasing if only the strict inequality holds true. Conversely, $V_i(t)$ is monotonically decreasing if and only if $-V_i(t)$ is monotonically increasing.

It is assumed that since the time dependency of the system revenue is accounted for with the unit availability function, then the revenue function, $V_i(t)$, can be represented as a time independent monotonically increasing function of the number of units. Thus,

$$(3.25) \quad V_i(t) = V_i.$$

However, the specific form of the revenue function will depend upon the particular system and its usage. The system user is expected to obtain the specific revenue function which is appropriate. This can be done either analytically, for some systems, or empirically, for others.

The present worth of the expected revenue obtained over a time duration T, is designated K_4.

$$(3.26) \quad K_4 = \int_0^T \sum_{i=0}^{N} V_i \, P_i(t) \exp(-rt) \, dt.$$

The summation extends over the range of units, from 0 to N, because even though at least 1 unit is usually required for a positive return, there may be some cases where a negative return results when 0 units are operational. The model is applicable in such cases by choosing V_0 negative.

The calculation of $P_i(t)$, the probability of exactly i units operating at time t, depends upon the specific spare configuration considered and will be left for the example problem of Chapter V.

Example Calculations

Consider the schematic illustration of Figure 3.4, in which $N=2$, $P=3$, $m_1=2$, $m_2=1$, and $m_3=1$.

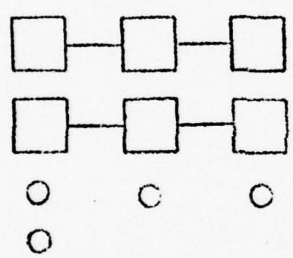


Figure 3.4 Specific System Configuration

The expected manufacturing cost is equivalent to its present worth since the units and their spare supplies are manufactured at time zero. Thus the present worth of the expected manufacturing costs is given by:

$$(3.27)\qquad K_1 = 2\sum_{j=1}^{3} a_j + \sum_{j=1}^{3} a_j m_j$$

$$= (2+m_1)a_1 + (2+m_2)a_2 + (2+m_3)a_3$$

$$= 4a_1 + 3a_2 + 3a_3.$$

For a fixed system configuration, the present worth of the total manufacturing or procurement costs is a linear function of the respective component procurement costs.

The expected repair cost for the system shown in Figure 3.4 (p. 40) is calculated below using Equation (3.11).

$$\text{(3.28)}\quad E(\text{Repair Cost}(t)) = \sum_{j=1}^{3} b_j \sum_{i=1}^{1+m_j} i\left(f_j^{(1)}(t)*g_j(t)\right)^{*i} * R_j^{(1)}(t)$$

$$\begin{aligned} &= b_1\{f_1^{(1)}*g_1*R_1^{(1)} + 2(f_1^{(1)}*g_1)^{*2}*R_1^{(1)} \\ &\qquad + 3(f_1^{(1)}*g_1)^{*3}*R_1^{(1)}\} \\ &+b_2\{f_2^{(1)}*g_2*R_2^{(1)} + 2(f_2^{(1)}*g_2)^{*2}*R_2^{(1)}\} \\ &+b_3\{f_3^{(1)}*g_3*R_3^{(1)} + 2(f_3^{(1)}*g_3)^{*2}*R_3^{(1)}\} \end{aligned}$$

For notational convenience, the time dependence of the failure, repair, and reliability density functions was not given in Equation (3.28).

The present worth of the expected repair cost for the system of Figure 3.4, with interest compounded continuously at a nominal annual rate r, over a time horizon T, is given below.

$$\text{(3.29)}\quad K_2 = \int_0^T \sum_{j=1}^{3} b_j \sum_{i=1}^{1+m_j} \left[i\left(f_j^{(1)}(t)*g_j(t)\right)^{*i} * R_j^{(1)}(t)\right] \exp(-rt)\, dt.$$

Assuming an exponential density function of the time to failure and time to repair, it is possible to obtain the closed form expression for the present worth of the expected repair/replacement costs

for the system of Figure 3.4(p. 40).

The specific density functions of the time to failure, time to repair, as well as the reliability function are given below.

(3.30) $f_j^{(1)}(t) = \beta_j \exp(-\beta_j t)$, where

$\beta_j = N\lambda_j$, for j=1, 2, and 3.

(3.31) $g_j(t) = \mu_j \exp(-\mu_j t)$, for j=1, 2, and 3.

(3.32) $R_j^{(1)}(t) = \exp(-\beta_j t)$, for j=1, 2, and 3.

Using the basic formulas of Table 3.1, the following convolution results are obtained.

(3.33) $$f_j^{(1)}(t) * g_j(t) = \frac{\beta_j\mu_j}{\beta_j - \mu_j}\{\exp(-\mu_j t) - \exp(-\beta_j t)\}$$

$$= \mu_j^2 \, t \exp(-\lambda_j t), \text{ if } \beta_j = \mu_j.$$

(3.34) $$(f_j^{(1)}(t) * g_j(t))^{*2} = \left(\frac{\beta_j\mu_j}{\beta_j - \mu_j}\right)^2 \left\{ t(\exp(-\mu_j t) + \exp(-\beta_j t)) - \frac{2(\exp(-\mu_j t) - \exp(-\beta_j t))}{(\beta_j - \mu_j)}\right\}$$

$$= (t^3/6) \exp(-\mu_j t), \text{ if } \beta_j = \mu_j.$$

TABLE 3.1 Convolved Exponential Functions

$\exp(-\mu t) * \exp(-\mu t)$	$t \exp(-\mu t)$
$\exp(-\mu t) * t \exp(-\mu t)$	$(t^2/2) \exp(-\mu t)$
$\exp(-\mu t) * t^n \exp(-\mu t)$	$(t^{n+1}/(n+1)!) \exp(-\mu t)$
$\exp(-\lambda t) * \exp(-\mu t)$	$(\exp(-\mu t) - \exp(-\lambda t))/(\lambda - \mu)$
$\exp(-\lambda t) * t \exp(-\mu t)$	$\frac{t \exp(-\mu t)}{(\lambda - \mu)} + \frac{(\exp(-\lambda t) - \exp(-\mu t))}{(\lambda - \mu)^2}$
$\exp(-\mu t) * t \exp(-\lambda t)$	$\frac{t \exp(-\lambda t)}{(\mu - \lambda)} + \frac{(\exp(-\mu t) - \exp(-\lambda t))}{(\mu - \lambda)^2}$
$\exp(-\lambda t) * t^2 \exp(-\mu t)$	$\frac{t^2 \exp(-\mu t)}{(\lambda - \mu)} - \frac{2t \exp(-\mu t)}{(\lambda - \mu)^2} + \frac{2 \exp(-\mu t)}{(\lambda - \mu)^3} - \frac{2 \exp(-\lambda t)}{(\lambda - \mu)^3}$
$\exp(-\mu t) * t^2 \exp(-\lambda t)$	$\frac{t^2 \exp(-\lambda t)}{(\mu - \lambda)} - \frac{2t \exp(-\lambda t)}{(\mu - \lambda)^2} + \frac{2 \exp(-\lambda t)}{(\mu - \lambda)^3} - \frac{2 \exp(-\mu t)}{(\mu - \lambda)^3}$

$$(3.35)\quad (f_j^{(1)}(t)*g_j(t))^{*3} = \left(\frac{\beta_j\mu_j}{\beta_j-\mu_j}\right)^3\{(t^2/2)(\exp(-\mu_jt) - \exp(-\beta_jt))$$

$$- \frac{t}{(\beta_j-\mu_j)}(3\exp(-\mu_jt) + 3\exp(-\beta_jt))$$

$$+ \frac{6}{(\beta_j-\mu_j)^2}(\exp(-\mu_jt) - \exp(-\beta_jt))\}$$

$$= (\mu_j^6/3!)\,(t^5/20)\exp(-\mu_jt), \text{ if } \beta_j=\mu_j.$$

The respective reliability convolutions are given below.

$$(3.36)\quad f_j^{(1)}(t)*g_j(t)*R_j^{(1)}(t) = \left(\frac{\beta_j\mu_j}{\beta_j-\mu_j}\right)\left\{\frac{\exp(-\mu_jt) - \exp(-\beta_jt)}{(\beta_j-\mu_j)}\right.$$

$$\left. - t\exp(-\beta_jt)\right\}$$

$$= \mu_j^2(t^2/2)\exp(-\mu_jt), \text{ if } \beta_j=\mu_j.$$

$$(3.37)\quad (f_j^{(1)}*g_j(t))^{*2}*R_j^{(1)}(t) =$$

$$\left(\frac{\beta_j\mu_j}{\beta_j-\mu_j}\right)^2\left\{\frac{3\exp(-\beta_jt) - 3\exp(-\mu_jt)}{(\beta_j-\mu_j)} + \frac{2t\exp(-\beta_jt) + t\exp(-\mu_jt)}{(\beta_j-\mu_j)} + (t^2/2)\exp(-\beta_jt)\right\}$$

$$= \mu_j^2(t^2/2)\exp(-\mu_jt), \text{ if } \mu_j=\beta_j.$$

$$(3.38) \quad (f_j^{(1)}(t)*g_j(t))^{*3} * R_j^{(1)}(t) = \left(\frac{\beta_j \mu_j}{\beta_j - \mu_j}\right)^3 \quad X$$

$$\left\{ \frac{10(\exp(-\mu_j t) - \exp(-\beta_j t))}{(\beta_j - \mu_j)^3} - \frac{(4t \exp(-\mu_j t) + 6t \exp(-\beta_j t))}{(\beta_j - \mu_j)^2} \right.$$

$$\left. + (1/2)\left(\frac{t(\exp(-\mu_j t) - 3t^2 \exp(-\beta_j t))}{(\beta_j - \mu_j)} - t^3 \frac{\exp(-\beta_j t)}{12} \right) \right\}$$

$$= \mu_j^6 \frac{t^6 \exp(-\mu_j t)}{720}, \text{ if } \beta_j = \mu_j.$$

The present worth of the expected repair/replacement costs is obtained by integrating the product of the expected repair costs and exp(-rt), where r is the continuous annual interest rate. Equation (3.29) contains such a calculation for the system of Figure 3.4 (p. 40).

Using Equation (3.28), together with the results derived in Equations (3.36-3.38), the expected repair cost for the system of Figure 3.4 is tabulated in Tables 3.2 and 3.3. Table 3.3 contains the expected repair cost equation when the system hazard rate, β_j (= $N\lambda_j$), equals the repair rate, μ_j, for j=1, 2, and 3.

Assuming $b_1=b_2=b_3=1$, the normalized present worth of the repair/replacement costs is calculated below. A comparison will be

TABLE 3.2 Expected Repair/Replacement Cost for System of Figure 3.4. Hazard Rate = β_j (= $N\lambda_j$), Repair Rate = μ_j. Component Replacement Cost = b_j, for j = 1, 2, and 3.

$$E\{\text{Repair Cost}(t)\} = \sum_{j=1}^{3} \sum_{i=1}^{1+m_j} i\left(f_j^{(1)}(t)*g_j(t)\right)^{*i} * R_j^{(1)}(t).$$

Component 1 (j=1)

$$b_1\left\{\left(\frac{\beta_1\mu_1}{\beta_1-\mu_1}\right)\left(\frac{\exp(-\mu t)-\exp(-\beta_1 t)}{(\beta_1-\mu_1)} - t\exp(-\beta_1 t)\right)\right.$$

$$+\,2\left(\frac{\beta_1\mu_1}{\beta_1-\mu_1}\right)^2\left(\frac{3\exp(-\beta_1 t)-3\exp(-\mu_1 t)}{(\beta_1-\mu_1)^2} + \frac{2t\exp(-\beta_1 t)+t\exp(-\mu_1 t)}{(\beta_1-\mu_1)} + \frac{t^2\exp(-\beta_1 t)}{2}\right)$$

$$+\,3\left(\frac{\beta_1\mu_1}{\beta_1-\mu_1}\right)^3\left(\frac{10\exp(-\mu_1 t)-10\exp(-\beta_1 t)}{(\beta_1-\mu_1)^3} - \frac{(4t\exp(-\mu_1 t)+6t\exp(-\beta_1 t))}{(\beta_1-\mu_1)^2}\right.$$

$$\left.\left.+\frac{(0.5\,t^2\exp(-\mu_1 t)-1.5\,t^2\exp(-\beta_1 t))}{(\beta_1-\mu_1)} - t^3\frac{\exp(-\beta_1 t)}{12}\right)\right\}$$

TABLE 3.2 (continued)

Component 2 (j=2)

$$b_2\left\{\left(\frac{\beta_2\mu_2}{\beta_2-\mu_2}\right)\left(\frac{\exp(-\mu_2 t)-\exp(-\beta_2 t)}{(\beta_2-\mu_2)}-t\exp(-\beta_2 t)\right)\right.$$

$$+\,2\left(\frac{\beta_2\mu_2}{\beta_2-\mu_2}\right)^2\left(\frac{3\exp(-\beta_2 t)-3\exp(-\mu_2 t)}{(\beta_2-\mu_2)^2}+\frac{2t\exp(-\beta_2 t)+t\exp(-\mu_2 t)}{(\beta_2-\mu_2)}\right.$$

$$\left.\left.+\,t^2\,\frac{\exp(-\beta_2 t)}{2}\right)\right\}$$

TABLE 3.2 (continued)

Component 3 (j=3)

$$b_3 \left\{ \left(\frac{\beta_3 \mu_3}{\beta_3 - \mu_3}\right) \left(\frac{\exp(-\mu_3 t) - \exp(-\beta_3 t)}{(\beta_3 - \mu_3)} - t \exp(-\beta_3 t) \right) + 2 \left(\frac{\beta_3 \mu_3}{\beta_3 - \mu_3}\right) \left(\frac{3 \exp(-\beta_3 t) - 3 \exp(-\mu_3 t)}{(\beta_3 - \mu_3)^2} + \frac{2t \exp(-\beta_3 t) + t \exp(-\mu_3 t)}{(\beta_3 - \mu_3)} + t^2 \frac{\exp(-\beta_3 t)}{2} \right) \right\}$$

TABLE 3.3 Expected Repair/Replacement Cost for System of Figure 3.4. Hazard Rate = Repair Rate(μ_j)

Component Replacement Cost = b_j, for j = 1, 2, and 3.

$$E\{\text{Repair Cost}(t)\} = \sum_{j=1}^{3} \sum_{i=1}^{1+m_j} i(f_j^{(1)}(t)*g_j(t))^{*i} * R_j^{(1)}(t).$$

Component 1 (j=1)	$b_1 \{ \mu_1^2 \frac{t^2}{2} \exp(-\mu_1 t) + 2 \frac{\mu_1^4 t^4}{24} \exp(-\mu_1 t) + 3 \frac{\mu_1^6 t^6}{720} \exp(-\mu_1 t) \}$
Component 2 (j=2)	$b_2 \{ \mu_2 \frac{t^2}{2} \exp(-\mu_2 t) + 2 \frac{\mu_2^4 t^4}{24} \exp(-\mu_2 t) \}$
Component 3 (j=3)	$b_3 \{ \mu_3 \frac{t^2}{2} \exp(-\mu_3 t) + 2 \frac{\mu_3^4 t^4}{24} \exp(-\mu_3 t) \}$

made among the present worth values for particular hazard and repair rates. In the case of exponential failure and repair densities, the analysis will be tabulated as a function of the system mean time between failure, and the mean time to repair, MTBF and MTTR, respectively. The system mean time between failure, for the j^{th} component type, is $1/\beta_j$, which equals $1/N\lambda_j$. Also, assuming equivalent hazard and repair rates for each component type, the present worth of the expected repair/replacement costs is calculated using the equations given in Table 3.3 (p. 49).

From Equation (3.29), the closed form expression for K_2 is:

$$
\begin{aligned}
(3.39)\qquad K_2 = &\int_0^T b_1 \sum_{i=1}^{3} i(f_1^{(1)}(t)*g_1(t))^{*i} * R_1^{(1)}(t)\exp(-rt)\,dt \\
+&\int_0^T b_2 \sum_{i=1}^{2} i(f_2^{(1)}(t)*g_2(t))^{*i} * R_2^{(1)}(t)\exp(-rt)\,dt \\
+&\int_0^T b_3 \sum_{i=1}^{2} i(f_3^{(1)}(t)*g_3(t))^{*i} * R_3^{(1)}(t)\exp(-rt)\,dt.
\end{aligned}
$$

Substituting the formulas of Table 3.3 into Equation (3.29), and normalizing the replacement cost coefficients, $b_j=1$, for $j=1$, 2, and 3, yields the following integral equation for the present worth of the repair/replacement costs.

$$(3.40) \qquad K_2 = \int_0^T \mu_1^{\,2} (t^2/2) \exp(-(\mu_1 + r)t)\, dt$$

$$+ \int_0^T \mu_2^{\,2} (t^2/2) \exp(-(\mu + r)t)\, dt$$

$$+ \int_0^T \mu_3^{\,2} (t^2/2) \exp(-(\mu_3 + r)t)\, dt$$

$$+ 2(\int_0^T \mu_1^{\,4} (t^4/6\cdot 4) \exp(-(\mu_1 + r)t)\, dt$$

$$+ 2(\int_0^T \mu_2^{\,4} (t^4/6\cdot 4)) \exp(-(\mu_2 + r)t)\, dt$$

$$+ 2(\int_0^T \mu_3^{\,4} (t^4/6\cdot 4)) \exp(-(\mu_3 + r)t)\, dt$$

$$+ 3 \int_0^T \mu_1^{\,6} (t^6 / 20\cdot 6\cdot 3!) \exp(-(\mu_1 + r)t)\, dt$$

Tables 3.4 and 3.5 contain listings of the normalized present worth of the repair/replacement costs, K_2, for the system of Figure 3.4, as a function of time, and the continuous interest rate. Annual time periods require the dimensions of the interest rate to be "percent per year". By "normalized present worth", is meant the present worth with cost coefficients, b_1, b_2, and b_3, of unity. Table 3.4 contains the values of K_2 with the MTBF and MTTR both equal to 1.00, while Table 3.5 contains the values when both the MTBF,

TABLE 3.4 Normalized Present Worth of Repair/Replacement Costs with MTBF=MTTR=1.0.

MTBF	MTTR
1.00	1.00

TIME IR	1	2	3	4	5	6	7	8	9	10
10	.24	1.13	2.41	3.75	4.93	5.85	6.50	6.92	7.19	7.34
11	.24	1.12	2.36	3.66	4.79	5.66	6.26	6.66	6.90	7.04
12	.24	1.10	2.31	3.57	4.65	5.47	6.04	6.41	6.63	6.75
13	.24	1.09	2.27	3.48	4.51	5.29	5.82	6.16	6.37	6.48
14	.24	1.07	2.22	3.39	4.38	5.12	5.62	5.93	6.12	6.23
15	.24	1.06	2.18	3.31	4.25	4.95	5.42	5.71	5.89	5.98
16	.23	1.04	2.14	3.23	4.13	4.79	5.23	5.50	5.66	5.75
17	.23	1.03	2.10	3.15	4.01	4.64	5.05	5.30	5.45	5.53
18	.23	1.02	2.06	3.07	3.90	4.49	4.88	5.11	5.25	5.32
19	.23	1.00	2.02	3.00	3.79	4.35	4.71	4.93	5.05	5.12
20	.23	.99	1.98	2.93	3.68	4.21	4.55	4.76	4.87	4.93

TABLE 3.5 Normalized Present Worth of Repair/Replacement Costs with MTBF=MTTR=10.0.

MTBF	MTTR
10.00	10.00

TIME / IR	1	2	3	4	5	6	7	8	9	10
10	.00	.03	.09	.18	.31	.47	.65	.86	1.08	1.31
11	.00	.03	.09	.18	.30	.45	.62	.81	1.02	1.23
12	.00	.03	.08	.17	.29	.43	.59	.77	.96	1.15
13	.00	.03	.08	.17	.28	.41	.56	.73	.90	1.07
14	.00	.03	.08	.16	.27	.39	.54	.69	.85	1.01
15	.00	.03	.08	.16	.26	.38	.51	.65	.80	.94
16	.00	.03	.08	.15	.25	.36	.49	.62	.75	.88
17	.00	.03	.08	.15	.24	.35	.47	.59	.71	.83
18	.00	.03	.07	.14	.23	.33	.44	.56	.67	.78
19	.00	.03	.07	.14	.22	.32	.42	.53	.63	.73
20	.00	.03	.07	.14	.22	.31	.40	.50	.60	.69

and MTTR equal 10.00[2]. They were calculated using Equation (3.40)

Examining the tables indicates that the normalized present worth of the repair/replacement costs is relatively insensitive to variations in the interest rate between 10% and 20% per period. Also noted is the fact that, for specified time horizons between 1 and 10 periods, the normalized present worth under the larger values of MTBF and MTTR is significantly less than that obtained with the smaller values of MTBF and MTTR. The reason for this is that with the larger values of MTBF and MTTR, there is a smaller probability of failing and completing the repair or replacement procedure within the specified time horizon. Consequently, the expected cost of the failure and repair completion is small, as is its present worth.

Example Calculation of Cannibalization Probability

Consider again the system shown in Figure 3.4 (p. 40), with 2 units, 3 components per unit, and 2 spares for the first component type, 1 spare for the second, and 1 spare for the third.

According to the original definition, $D_1(t)$ is the probability of cannibalizing a unit because of a failure of the first component type. In the example of Figure 3.4, a unit is cannibalized due to a failure of type 1 only if the third failure of component type 1 occurs before the smaller of the second failure times of components 2 and 3. Specifically,

---

[2]Appendix II contains tables for the present worth calculations of K_2, for a system MTBF of 1.00, and an MTTR between 0.10 and 0.33.

(3.41) $D_1(t) = Pr\{ t_1^{(3)} < Min(t_2^{(2)}, t_3^{(2)}) \}$.

(3.42) $D_2(t) = Pr\{ t_2^{(2)} < Min(t_1^{(3)}, t_3^{(2)}) \}$.

(3.43) $D_3(t) = Pr\{ t_3^{(2)} < Min(t_1^{(3)}, t_2^{(2)}) \}$.

Here, $t_1^{(3)}$ is the third failure time of component type 1, $t_2^{(2)}$ is the second failure time of component type 2, and $t_3^{(2)}$ is the second failure time of component type 3. The probability that a particular order statistic is less than the smaller of two other order statistics is the probability that it is smaller than both of the other order statistics. Thus the expressions for $D_j(t)$, for $j=1$, 2, and 3, are as follows:

(3.44) $D_1(t) = Pr\{t_1^{(3)} < t_2^{(2)}, \text{ and } t_1^{(3)} < t_3^{(2)}\}$

(3.45) $D_2(t) = Pr\{t_2^{(2)} < t_1^{(3)}, \text{ and } t_2^{(2)} < t_3^{(2)}\}$

(3.46) $D_3(t) = Pr\{t_3^{(2)} < t_1^{(3)}, \text{ and } t_3^{(2)} < t_2^{(2)}\}$

Since the components are assumed to fail independently according to the specific failure density function, $f_j(t)$, then any order statistics of different component types must be independent. Consequently, the joint events expressed in Equations (3.44-3.46) are

independent.

$$(3.47) \quad D_1(t) = \Pr\{t_1^{(3)} < t_2^{(2)}\} \times \Pr\{t_1^{(3)} < t_3^{(2)}\}$$

$$(3.48) \quad D_2(t) = \Pr\{t_2^{(2)} < t_1^{(3)}\} \times \Pr\{t_2^{(2)} < t_3^{(2)}\}$$

$$(3.49) \quad D_3(t) = \Pr\{t_3^{(2)} < t_1^{(3)}\} \times \Pr\{t_3^{(2)} < t_2^{(2)}\}$$

Using the assumption of independence among each of the respective ordered times to failure, $t_1^{(3)}$, $t_2^{(2)}$, and $t_3^{(2)}$, the joint density function of the ordered failure times is the product of the marginal densities. The cannibalization probability expressions given in Equations (3.47-3.49) are expressed as the product of the integrals of the joint density functions.

$$(3.50) \quad D_1(t) = \int_0^t \int_0^y h_1^{(3)}(x)\, h_2^{(2)}(y)\, dx\, dy \quad \times$$
$$\int_0^t \int_0^y h_1^{(3)}(x)\, h_3^{(2)}(y)\, dx\, dy.$$

$$(3.51) \quad D_2(t) = \int_0^t \int_0^y h_2^{(2)}(x)\, h_1^{(3)}(y)\, dx\, dy \quad \times$$
$$\int_0^t \int_0^y h_2^{(2)}(x)\, h_3^{(2)}(y)\, dx\, dy.$$

$$(3.52) \quad D_3(t) = \int_0^t \int_0^y h_3^{(2)}(x)\, h_1^{(3)}(y)\, dx\, dy \quad X$$
$$\int_0^t \int_0^y h_3^{(2)}(x)\, h_2^{(2)}(y)\, dx\, dy.$$

The above results are consistent with the general expression for $D_j(t)$ given in Equation (3.19), even though they were derived independently.

For the case of an infinite replenishment rate, the marginal density functions of the ordered time to failure for each component type are equivalent to the respective ordered failure time density functions, $f_1^{(3)}(t)$, $f_2^{(2)}(t)$, and $f_3^{(2)}(t)$.

$$(3.53) \quad h_1^{(3)}(t) = f_1^{(3)}(t).$$

$$(3.54) \quad h_2^{(2)}(t) = f_2^{(2)}(t).$$

$$(3.55) \quad h_3^{(2)}(t) = f_3^{(2)}(t).$$

The equations for the density function of the ordered time to failure are given below.

$$(3.56) \quad f_1^{(3)}(t) = \binom{4}{2,1,1} F_1(t)^2 \quad f_1(t) \quad (1-F_1(t)).$$

$$(3.57) \quad f_2^{(2)}(t) = \binom{2}{1,1,0} F_2(t) \quad f_2(t) \quad (1-F_2(t))^0.$$

$$(3.58) \quad f_3^{(2)}(t) = \binom{2}{1,1,0} F_3(t) \quad f_3(t) \quad (1-F_3(t))^0.$$

For the exponential density function of time to failure, with the hazard rate for each component type, j, given by λ_j, the marginal density functions of ordered times to failure are:

$$(3.59) \quad h_1^{(3)}(t) = 12\ \lambda_1\ (1-\exp(-\lambda_1 t))^2\ \exp(-2\lambda_1 t)$$

$$(3.60) \quad h_2^{(2)}(t) = 2\ \lambda_2\ (1-\exp(-\lambda_2 t))\ \exp(-1\lambda_2 t)$$

$$(3.61) \quad h_3^{(2)}(t) = 2\ \lambda_3\ (1-\exp(-\lambda_3 t))\ \exp(-1\lambda_3 t)$$

Substituting the expressions given in Equations (3.59-3.61) into the equations for the component cannibalization probability, (3.50-3.52), and performing the respective integrations, yields the time dependent component cannibalization probability, $D_j(t)$, for j=1, 2, and 3. These expressions for the time dependent cannibalization probability are plotted in Figures 3.5 through 3.10, as a function of time. Note that they do not integrate to one because they are not plotted as probability density functions; $D_j(t)$ represents the time dependent probability that the j^{th} component type caused a cannibalization. Each set of graphs contains the respective cannibalization probability function for selected values of the MTBF for the system configuration of Figure 3.4 (p. 40).

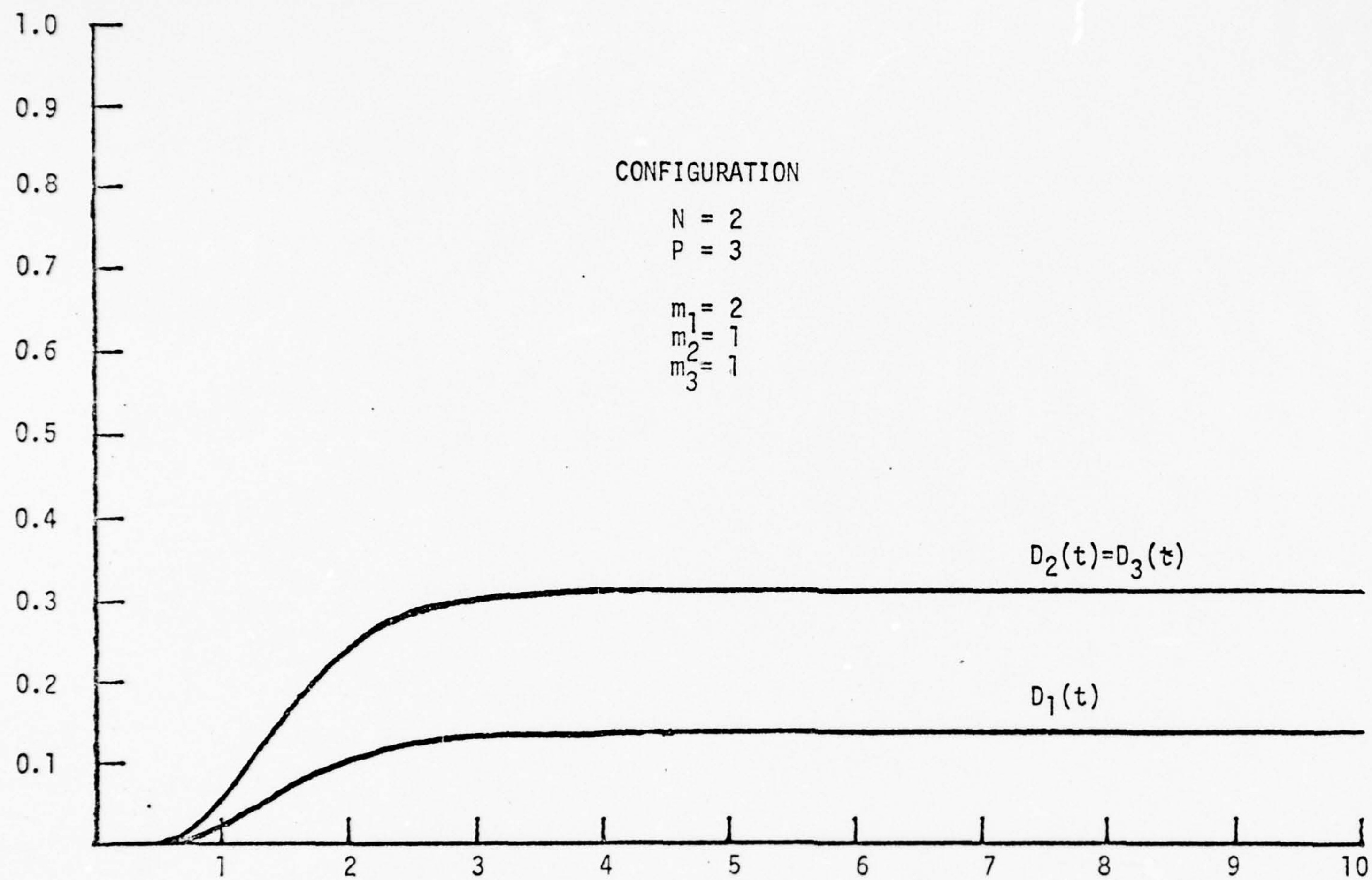


Figure 3.5 Component Cannibalization Probability, $D_j(t)$, versus Time, with MTBF(1)=MTBF(2)=MTBF(3)=1.0.

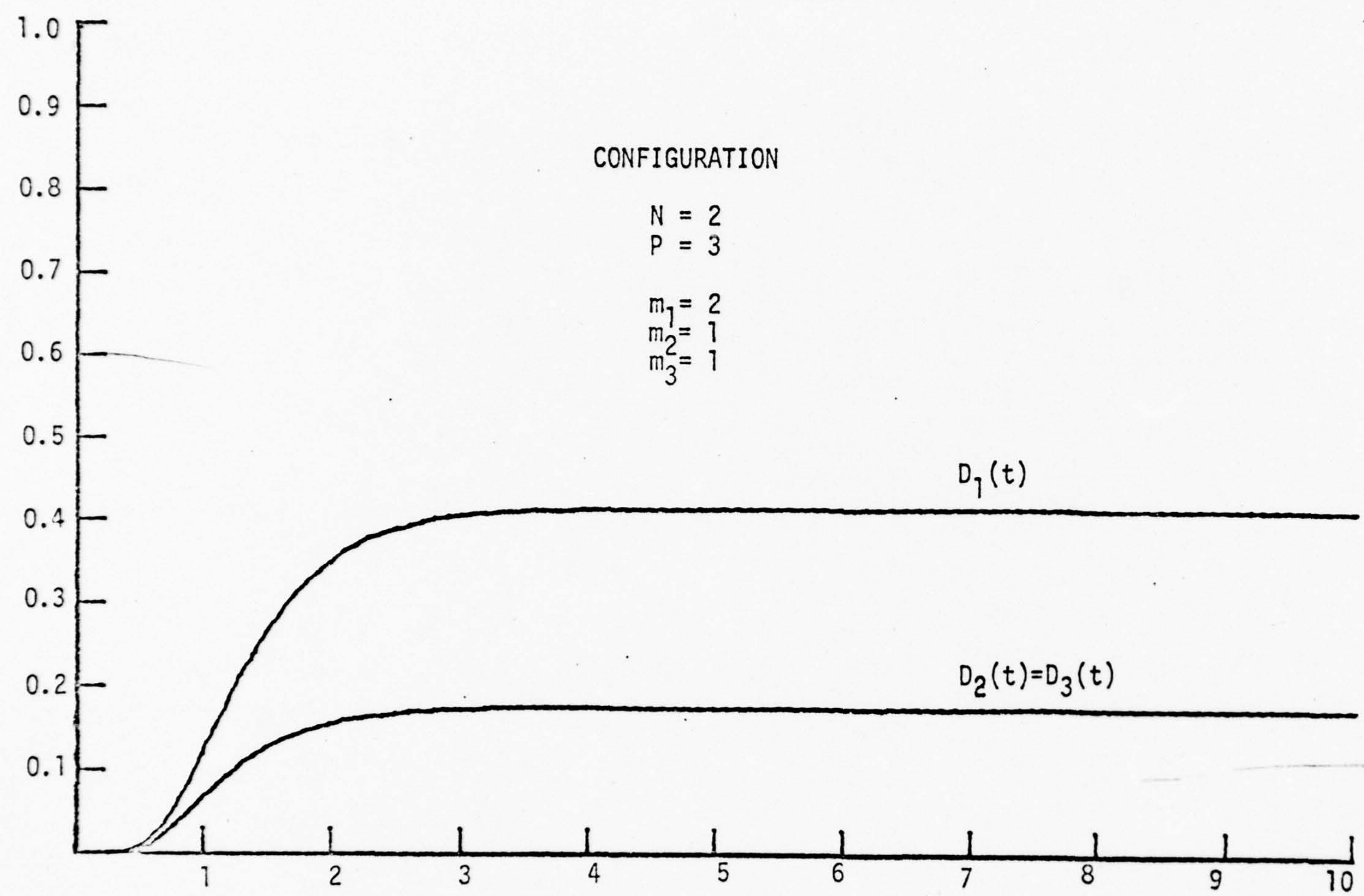


Figure 3.6 Component Cannibalization Probability, $D_j(t)$, versus Time, with MTBF(1)=0.5, MTBF(2)=MTBF(3)=1.0.

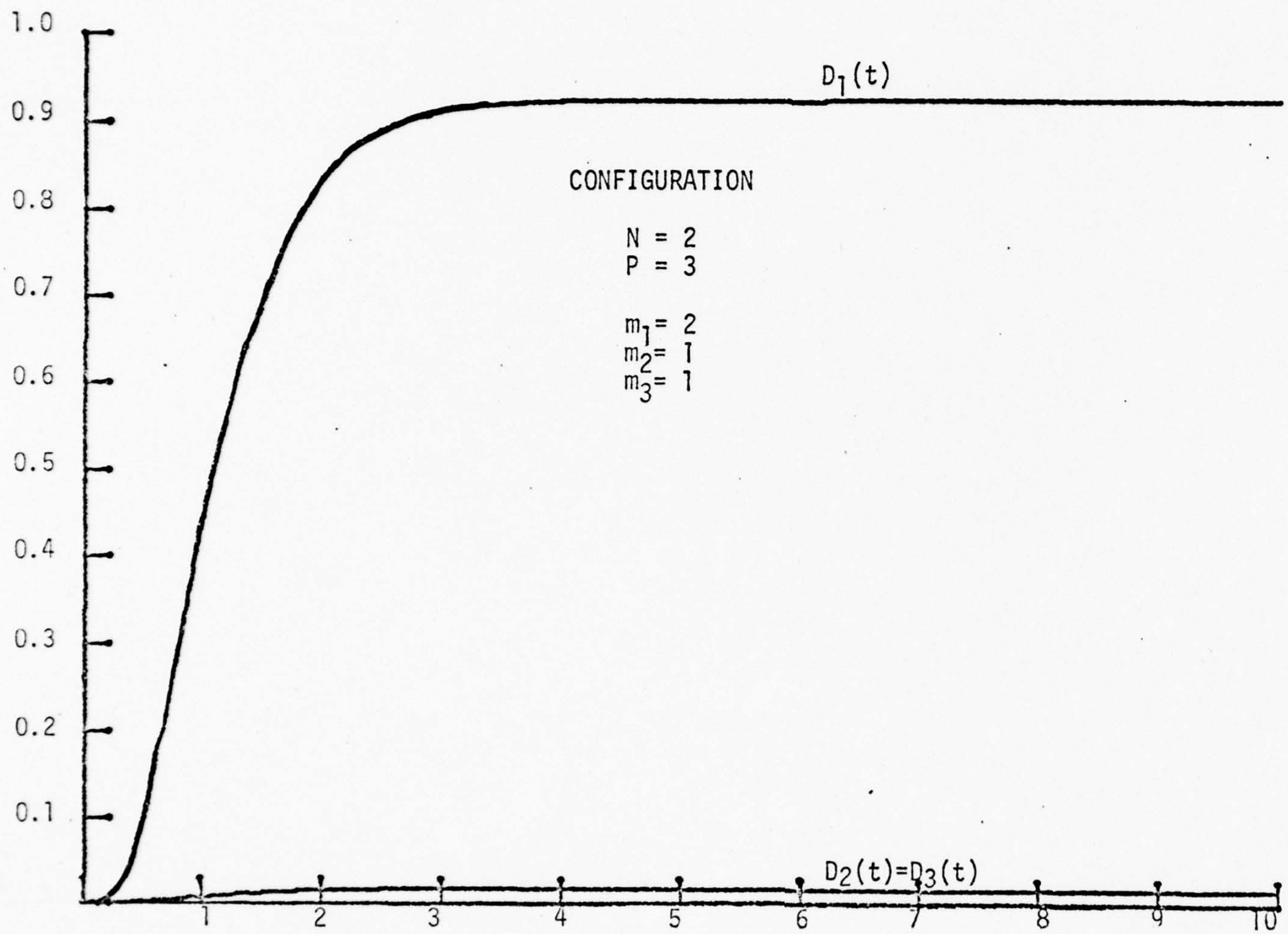


Figure 3.7 Component Cannibalization Probability, $D_j(t)$, versus Time, with MTBF(1)=0.10, MTBF(2)=MTBF(3)=1.0.

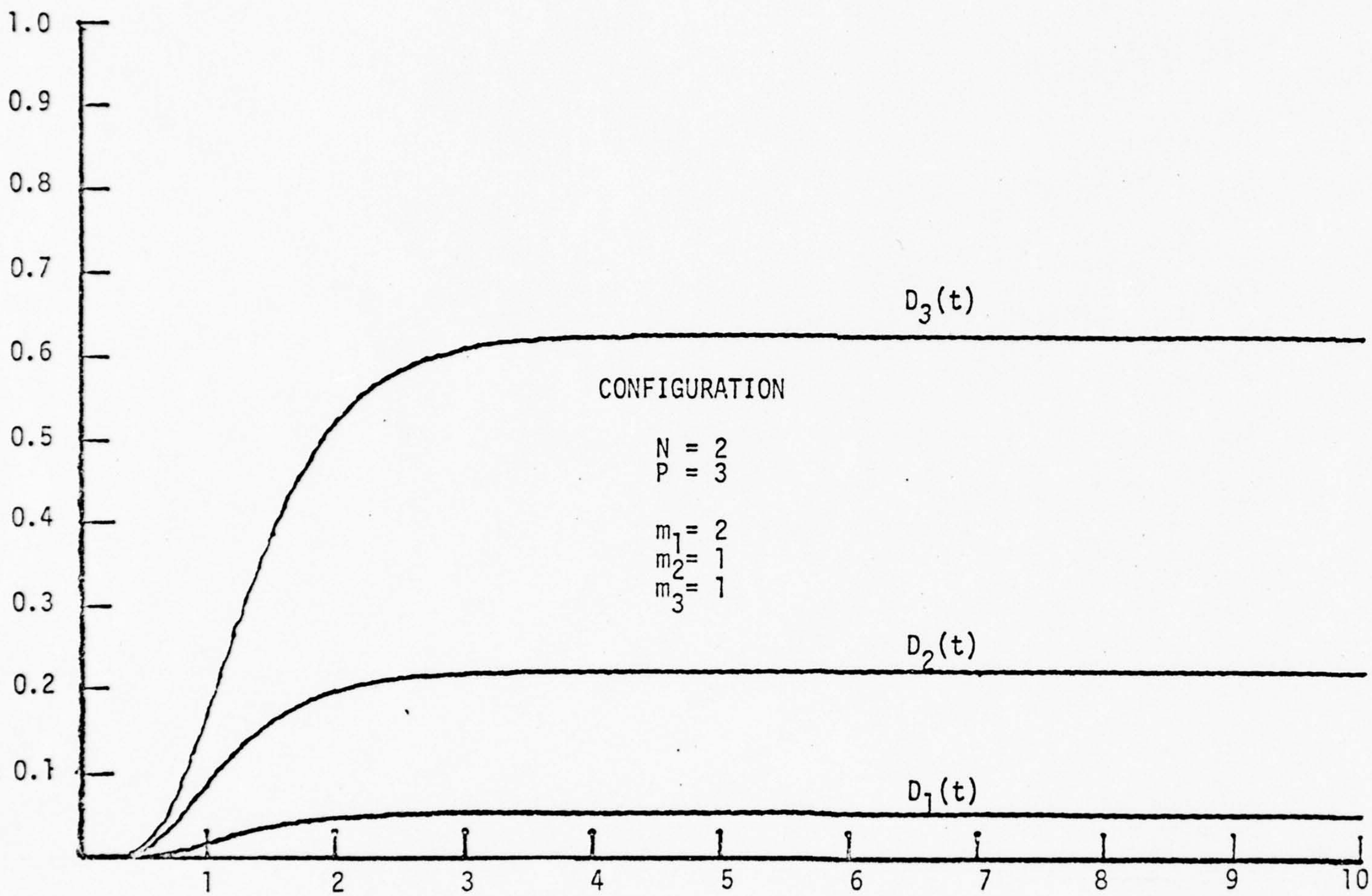


Figure 3.8 Component Cannibalization Probability, $D_j(t)$, versus Time, with MTBF(1)=MTBF(2)=1.0, MTBF(3)=0.5

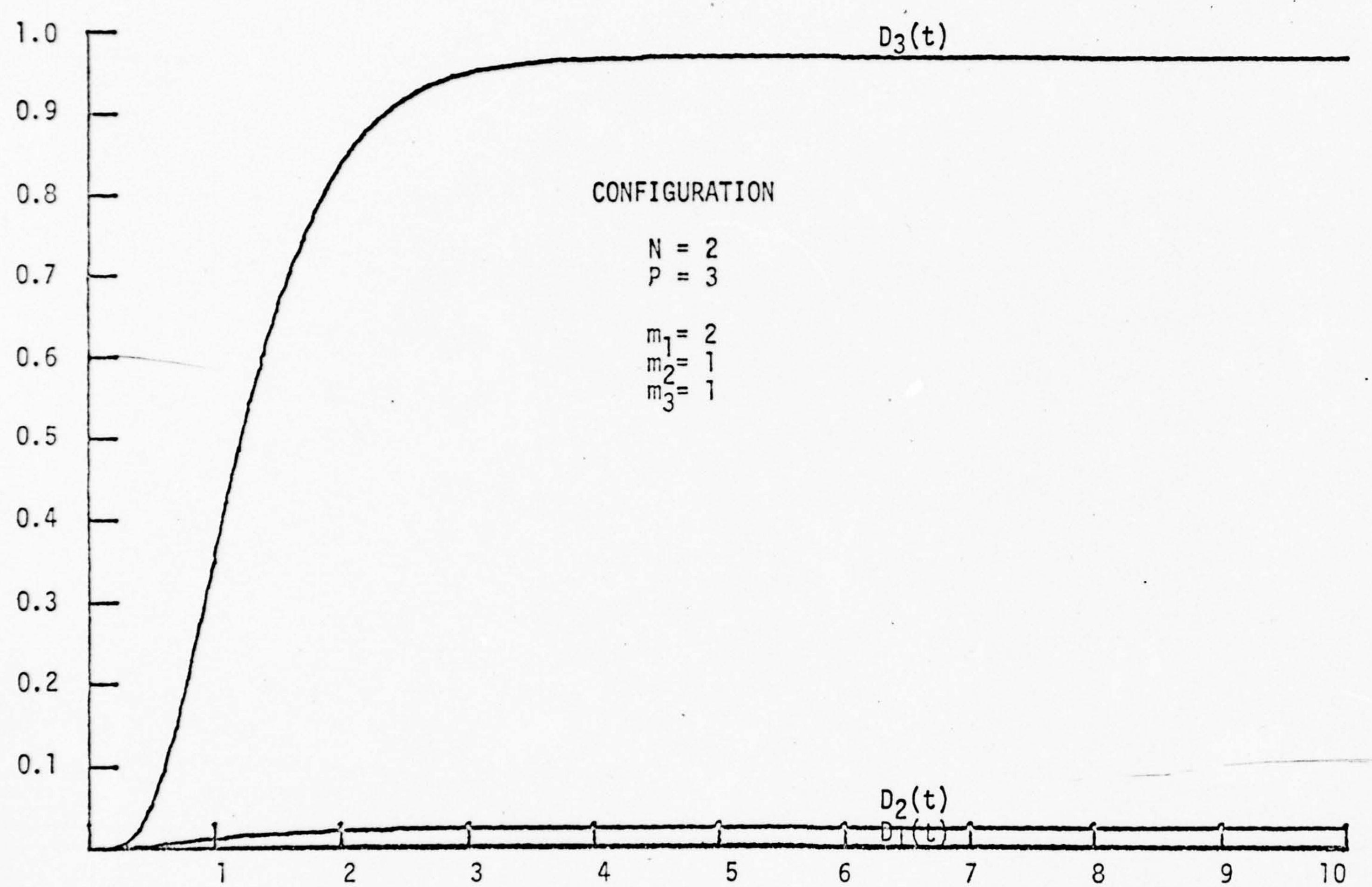


Figure 3.9 Component Cannibalization Probability, $D_j(t)$, versus Time, with MTBF(1)=MTBF(2)=1.0, MTBF(3)=0.10.

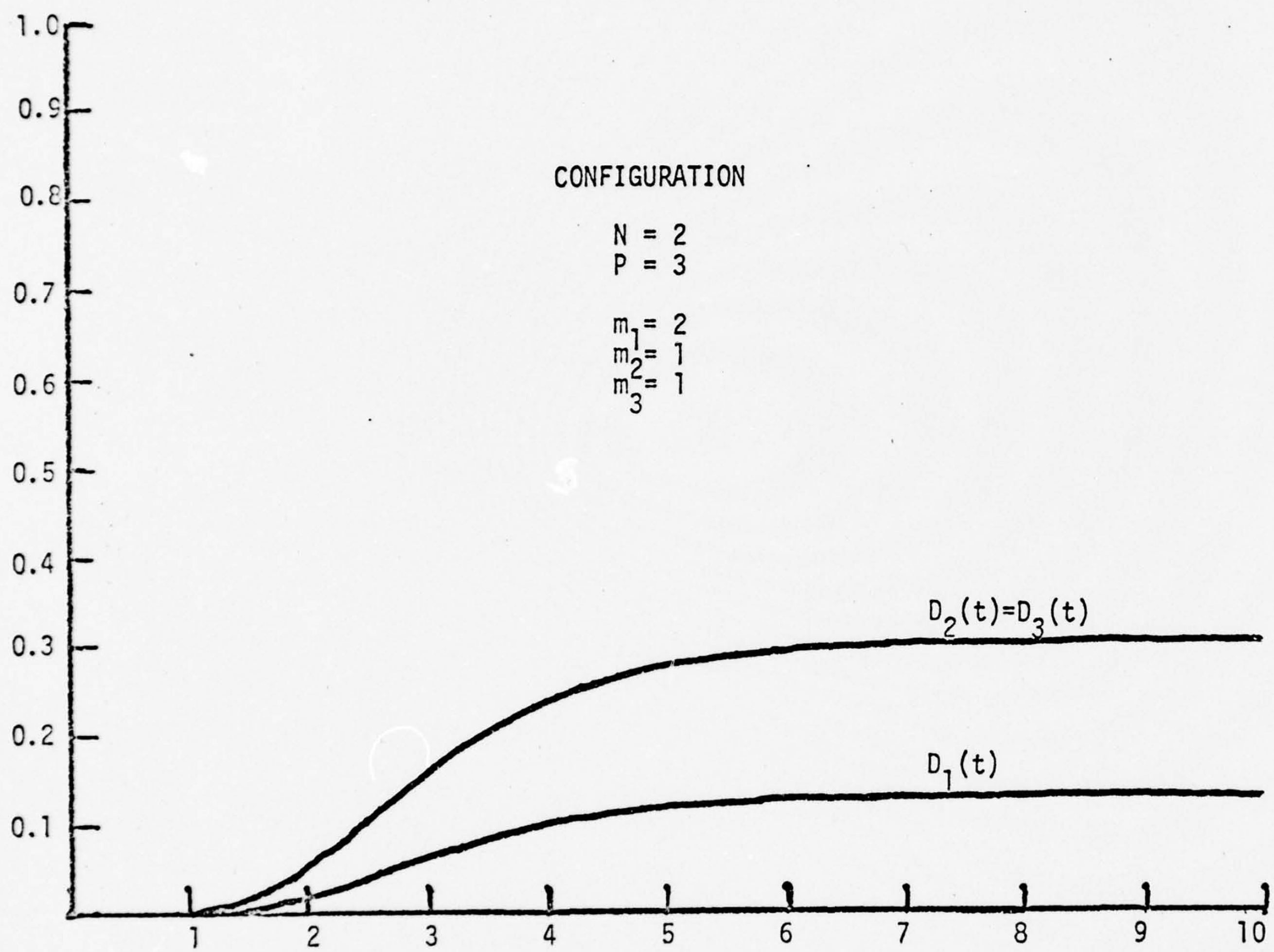


Figure 3.10 Component Cannibalization Probability, $D_j(t)$, versus Time, with MTBF(1)=MTBF(2)=MTBF(3)=2.0.

By varying the magnitude of the respective failure rates, (reciprocal MTBF's) the time required to reach a steady state probability level varies. This is seen by comparing Figures 3.5 and 3.10. In Figure 3.5 (p. 59), each component type has an MTBF of 1.0 and the steady state probability level is reached within approximately 3 time periods. In Figure 3.10 (p. 64), however, the respective component MTBF values are all 2.0 and the steady state probability level is reached after 6 time periods.

The steady state cannibalization probability is obtained by evaluating the cannibalization probability equations with the upper limit of the outer integrals set at infinity.

Table 3.6 contains selected steady state cannibalization probability values as a function of the respective component hazard rates, for the system configuration of Figure 3.4, having N=2, P=3, $m_1=2$, $m_2=1$, and $m_3=1$. It also contains the probability of no cannibalization, $1-D_1-D_2-D_3$. The tabulated values indicate that it is primarily the relative magnitude of the hazard rate, rather than the absolute value, which determines the respective cannibalization probability.[3] For example, when the respective hazard rates are all increased or decreased by the same relative magnitude, the corresponding cannibalization probability does not change. This is seen when all of the hazard rates are equal to 1, or 100.

---

[3] Appendix III contains the steady state probability derivation.

TABLE 3.6 Steady State Cannibalization Probability as a Function of Component Hazard Rate.

λ_1	λ_2	λ_3	D_1	D_2	D_3	$1-D_1-D_2-D_3$
1	1	1	0.138	0.314	0.314	0.234
1	1	2	0.055	0.165	0.629	0.151
1	2	1	0.055	0.629	0.165	0.151
2	1	1	0.417	0.177	0.177	0.229
1	1	5	0.009	0.048	0.901	0.042
1	5	1	0.009	0.901	0.048	0.042
5	1	1	0.793	0.055	0.055	0.097
1	1	10	0.002	0.015	0.971	0.012
1	10	1	0.002	0.971	0.015	0.012
100	100	100	0.138	0.314	0.314	0.234

Discussion and Conclusions

This chapter has outlined the stochastic description of cannibalization as a maintainability technique and contains a specific operational cost model. Only those costs which were directly related to the basic system configuration were considered; specifically included were the manufacturing or procurement costs, the repair or replacement costs, and the cannibalization costs. A revenue or return function was included in the model as a negative cost.

An example calculation was performed for a specific system configuration and each of the costs were analyzed separately on a present worth basis. Plots of the cannibalization probability as a function of time were given and its limiting steady state behavior was found to be only a function of the relative magnitude of the respective component hazard rates.

Also included was a discussion of the important factors considered in the system revenue/return function. No specific analytic expression was given for the revenue/return function because the return to the system user is a function of the specific application required, as well as the usage environment encountered.

Chapter IV contains a discussion of specific system constraints and the example calculations required for a constraint on spare adequacy.

CHAPTER IV

SYSTEM CONSTRAINTS

Introduction

Since sparing allocation is an integer process, i.e. there can only be an integer number of spares allocated to any component type, the classical methods of constrained optimization, such as the Jacobian method and the Lagrangian method [43,50], are not applicable since they assume a continuous domain of spare component types.

Under the formulation specified in Chapter I, in order for a specific spare allocation to be optimal, it must result in a minimum total operational cost and, in some cases, satisfy one or more constraints. If the constraints are applicable, they are usually imposed by the user during the system's design stage. The constraints are usually in the form of performance requirements, or limitations on available resources.

Performance constraints are generally, but not necessarily, formulated using inequalities of the form "greater than or equal to ($\geq$)", while resource constraints are usually formulated using "less than or equal to ($\leq$)" inequalities.

Typical performance constraints are minimum requirements on a system's reliability over a specific time horizon, or a requirement on its interval availability over the same time horizon. In this

context, the reliability is defined as the probability that the system operates over some pre-selected time interval, while the interval availability, as defined by Rau [34], is the expected fraction of an interval of specified length that the system is in an operable state. Another performance constraint, which will be discussed in detail, is a constraint on the probability of spare adequacy over a specified time horizon. This constraint requires that the probability of experiencing a critical spare shortage before some specified time be strictly less than a specified value, e.g., $1-\alpha_0$. By "critical spare shortage" is meant an inventory depletion which precludes further system operation. Obviously, this performance constraint can be expressed conversely as a requirement that the probability of experiencing no critical spare shortages be at least as large as α_0, the complement of the previously specified constraint value.

Some obvious resource constraints are limitations on the total weight or volume of the spare components; another is a limitation on the total dollar investment in spares. The resource constraints appropriate to this sparing allocation problem can be quantitatively described, in general, as follows:

Assume the maximum allowable consumption of the k^{th} resource, e.g. weight, volume, or available funds, is Q_k, then the feasible solution space of spare component allocations, $(m_1, m_2, \ldots, m_P)$, belongs to the set L, where $L = \{m_j: \sum_{j=1}^{P} d_{kj}(m_j) \leq Q_k,\ k = 1,2,..S\}$.

The function $d_{kj}(m_j)$ yields the quantity of the k^{th} resource consumed by the allocation of m_j spares for the j^{th} component type, and the index k varies from 1 to S for constraints on S total resources.

Under the required resource constraint, the problem is to find the allocation of spares, $(m_1, m_2, \ldots, m_p)$, which minimizes the appropriate cost function, and belongs to the set L.

Until now, nothing has been said about the effects of a finite time horizon on the optimal spare allocation. Obviously, the length of time the system is expected to be operating is significant in determining the optimal spare allocation. Certain types of constraints are not meaningful for certain applications; for example, in the analysis of a tactical weapons system, the time horizon may be very short, consequently, the use of a steady state availability constraint is obviously inappropriate; in fact it is meaningless unless an infinite number of replacements are permitted for a particular class of components, requiring an allocation of an infinite number of spares for that component type.

It became necessary to develop a realistic constraint on the spares which was responsive to changes in the system's expected usage life, T, as well as to changes in the number of fielded units, N. An appropriate performance constraint applicable over a finite time horizon is a constraint on the probability of experiencing a critical spare shortage for any of the component types.

Constraint on Spare Shortages

It is necessary to obtain the minimum number of spares required in order to meet a specific constraint on the probability that there will be no critical spare shortage for any of the P component types. One such constraint is the requirement that the probability that no critical spare shortages occur for any of the P component types be at least as large as some user specified value, say α_0. A critical spare shortage was defined previously as an inventory depletion which precludes further system operation. Specifically, system operation terminates when more than $(N+m_j-1)$ failures of the j^{th} component type occur, for $j = 1, 2, \ldots, P$. Under cannibalization, as many as $(N-1)$ components may be added to the spare inventory of any one component type.

Mathematically, this constraint is defined as follows:

$$(4.1)\quad \Pr\left\{\begin{array}{c}\text{no critical spare shortage}\\ \text{is experienced for any of the } P \text{ component}\\ \text{types during the interval } [0,T]\end{array}\right\} \geq \alpha_0 .$$

Define M_j as the maximum number of possible replacements of the j^{th} component type. If cannibalization is permitted, $M_j = N+m_j-1$, otherwise $M_j = m_j$. Assuming component failures occur in accordance with the Poisson distribution, with mean time between failures, $1/\lambda_j$, and assuming a finite time horizon, T, the probability of experiencing more than M_j failures of the j^{th} component type is no greater than the probability of experiencing more than m_j failures, with $N \geq 1$.

This implies that it is easier to protect against critical spare shortages when cannibalization is permitted than when it is not, since the system can tolerate as many as $(N+m_j-1)$ failures rather than just m_j failures for the j^{th} component type.

It may seem superfluous to impose a constraint on spare adequacy, when the cannibalization process itself is used as a tool for preventing critical spare shortages. However, the definition of a critical spare shortage was a spare inventory depletion which prevented further system operation. Under cannibalization, the point at which system operation terminates is extended because of the permissible use of cannibalized components to repair impending component failures.

For any given component type, j, the probability that no more than M_j spares are required during the interval $[0,T]$ is just the probability that the $(M_j+1)^{th}$ failure time exceeds T. If $h_j^{(M_j)}(t)$ is the marginal density function of the time to the M_j^{th} failure of the j^{th} component type, then the probability that M_j spares are adequate is just the probability that there will be no more than M_j failures in $[0,T]$. Alternatively, this is just the probability that the $(M_j+1)^{th}$ failure time exceeds T. Thus the constraint is written as follows:

$$(4.2) \quad \Pr\{(M_j+1)^{th} \text{ failure time} > T\} = \int_T^{\infty} h_j^{(M_j+1)}(t)\, dt.$$

With P statistically independent components per unit, it is necessary that the $(M_j+1)^{th}$ failure time exceed T for all component

types. Thus the constraint becomes:

$$(4.3) \qquad \prod_{j=1}^{P} \int_{T}^{\infty} h_j^{(M_j+1)}(t)\, dt \geq \alpha_0 .$$

If there are N independent and identical units, the effective hazard rate for the j^{th} component type is $N\lambda_j$, where λ_j is the hazard rate of a single component of the j^{th} type.

Assuming Poisson failures, the times between failures are exponentially distributed. With exponential failure times and instantaneous replacement, the probability density function of the sum of (M_j+1) independent and identically distributed random variables, $h_j^{(M_j+1)}(t)$, is the gamma density function.

$$(4.4) \qquad h_j^{(M_j+1)}(t) = N\lambda_j (N\lambda_j t)^{M_j} \ \exp(-N\lambda_j t) \ / \ M_j!$$

The instantaneous replacement assumption is justified if the mean time to replace a particular component type is significantly less than its mean time between failures. Under the instantaneous replacement assumption, it is not necessary to include the times required to perform the replacements in the marginal density function of the time to the $(M_j+1)^{th}$ failure of the j^{th} type.

Integrating Equation (4.4) by parts yields:

$$(4.5) \qquad \int_{T}^{\infty} (N\lambda_j)\ (N\lambda_j t)^{M_j} \frac{\exp(-N\lambda_j t)}{M_j!}\, dt = \sum_{k=0}^{M_j} (N\lambda_j T)^k \frac{\exp(-N\lambda_j T)}{k!} .$$

Substituting the result of Equation (4.5) into Equation (4.3) yields the complete constraint.

$$(4.6)\quad \prod_{j=1}^{P} \sum_{k=0}^{M_j} (N\lambda_j T)^k \frac{\exp(-N\lambda_j T)}{k!} \geq \alpha_o,$$

where α_o is the minimum probability of spare adequacy, as specified by the system user, and $M_j = m_j$ when cannibalization is not permitted, or $M_j = N + m_j - 1$, when it is.

The level of protection against critical spare shortages for the j^{th} component type is the cumulative Poisson sum given in Equation (4.5). The constraint, as written in Equation (4.6) is the product of independent cumulative Poisson densities, each with mean, $N\lambda_j T$.

If one does not impose the requirement that any one component type has a higher priority for protection against critical spare shortages than other component types, and if there are no restrictions on the maximum number of spares allowed for any component type, then one can allocate spares for each of the P component types in accordance with Equation (4.6) as follows:

1. Starting with the first component type, solve for M_1 by setting $M_2, M_3, \ldots, M_P = \infty$. This forces the Poisson summation over components 2 through P to unity, and one need only find the smallest value of M_1 which satisfies:

$$(4.7)\quad \sum_{k=0}^{M_1} (N\lambda_1 T)^k \frac{\exp(-N\lambda_1 T)}{k!} \geq \alpha_o.$$

This can be calculated directly, or by using a table of the Cumulative Poisson Distribution Function.

2. Next, determine the smallest value of M_2 which satisfies:

$$\sum_{k=0}^{M_2} (N\lambda_2 T)^k \frac{\exp(-N\lambda_2 T)}{k!} \geq \alpha_0. \tag{4.8}$$

3. Continue this procedure for the remaining components. The minimum number of spares provisioned for the P^{th} *component type will satisfy:*

$$\sum_{k=0}^{M_P} (N\lambda_P T)^k \frac{\exp(-N\lambda_P T)}{k!} \geq \alpha_0. \tag{4.9}$$

This procedure is used to calculate a lower bound on the required number of spares for each component type. The allocation achieved will not necessarily satisfy the constraint of Equation (4.6), because the maximum of each Poisson summation is 1.0 when M_j equals infinity; thus the product over all P component types will be at most 1.0. (For almost all applications, it will be significantly less than 1.0.) However, if an exhaustive search is to be performed over the total costs associated with each allocation, then this procedure will yield the starting allocations over which the search should be conducted. Then, for each allocation, the product of the respective Poisson sums can be calculated, in accordance with Equation (4.6), to eliminate those allocations which do

not satisfy the constraint given in Equation (4.6).

The outlined procedure can be performed in any order, i.e. starting with any component type. However, it is only applicable if no component type has a higher priority for spare adequacy than other component types.

If one particular component type, say type "q", requires a spare sufficiency level of α_q for example, then M_q should be calculated as the smallest integer which satisfies:

$$(4.10) \quad \sum_{k=0}^{M_q} (N\lambda_q T)^k \frac{\exp(-N\lambda_q T)}{k!} \geq \alpha_q .$$

The remaining minimum spare allocations are calculated in accordance with Equations (4.7) through (4.9).

If all of the components do not have distinct MTBF's, for example if two or more of the components have equivalent MTBF's, then they will necessarily have identical Poisson mean values, and will be allocated the same number of spares initially.

There may be some situations in which the spare adequacy constraint given in Equation (4.1) is further restricted with the requirement that each of the component types must have the same level of protection against critical spare shortages. With such a restriction, the constraint given in Equation (4.6) becomes:

$$(4.11) \quad \sum_{k=0}^{M_j} (N\lambda_j T)^k \frac{\exp(-N\lambda_j T)}{k!} \geq (\alpha_0)^{1/P}, \text{ for } j = 1, \ldots, P,$$

where $M_j = m_j$, when cannibalization is not permitted, and $M_j = (N+m_j-1)$, when it is. Thus, the spare adequacy probability is partitioned

equally among the P component types.

The equal partitioning is justified as long as no component type has a higher priority for spare sufficiency than any other component type. If there were an additional requirement that the q^{th} component type have a spare adequacy probability in excess of α_q, then the single constraint of Equation (4.11) becomes the multiple constraint:

$$(4.12) \quad \sum_{k=0}^{M_q} (N\lambda_q T)^k \frac{\exp(-N\lambda_q T)}{k!} \geq \alpha_q, \text{ and}$$

$$\sum_{k=0}^{M_j} (N\lambda_j T)^k \frac{\exp(-N\lambda_j T)}{k!} \geq (\alpha_o/\alpha_q)^{1/(P-1)}, \text{ for } \begin{matrix} j = 1,\ldots,P. \\ j \neq q. \end{matrix}$$

Tables 4.1 through 4.5 contain the minimum spare allocations in order to meet a minimum probability of spare adequacy, α_o, in excess of 0.70, 0.75, 0.80, 0.85, and 0.90, for various values of N, MTBF, and T. Each table lists the minimum spare allocations necessary under both conditions: when cannibalization is permitted, and when it is not. The values tabulated are the minimum number of spares required, m_j, for each component type, j, j = 1, 2, and 3, calculated in accordance with Equations (4.7) through (4.9). The spare allocations are the same in Tables 4.4 and 4.5, because the respective values of $N\lambda_j T$ are the same in both tables.

The tabular solutions listed are relevant in this application because they are dependent upon both the time horizon, and the failure rates of each of the respective component types.

TABLE 4.1 Minimum Spare Allocations for a 2 Unit System with a Time Horizon of 5 Periods.

UNITS-N	2
TIME--T	5.00

COMP TYPE	MTBF	MINIMUM PROBABILITY OF SPARE ADEQUACY 0.70	0.75	0.80	0.85	0.90
CANNIBALIZATION NOT PERMITTED						
1	1.0	12	12	13	13	14
2	1.0	12	12	13	13	14
3	1.0	12	12	13	13	14
CANNIBALIZATION PERMITTED						
1	1.0	11	11	12	12	13
2	1.0	11	11	12	12	13
3	1.0	11	11	12	12	13

TABLE 4.2 Minimum Spare Allocations for a 2 Unit System with a Time Horizon of 10 Periods.

UNITS-N	2
TIME--T	10.00

COMP TYPE	MTBF	MINIMUM PROBABILITY OF SPARE ADEQUACY 0.70	0.75	0.80	0.85	0.90
CANNIBALIZATION NOT PERMITTED						
1	1.0	22	23	24	25	26
2	1.0	22	23	24	25	26
3	1.0	22	23	24	25	26
CANNIBALIZATION PERMITTED						
1	1.0	21	22	23	24	25
2	1.0	21	22	23	24	25
3	1.0	21	22	23	24	25

TABLE 4.3 Minimum Spare Allocations for a 2 Unit System with a Time Horizon of 10 Periods. (Distinct MTBF Values).

UNITS-N	2
TIME--T	10.00

COMP TYPE	MTBF	0.70	0.75	0.80	0.85	0.90
		MINIMUM PROBABILITY OF SPARE ADEQUACY				
		CANNIBALIZATION NOT PERMITTED				
1	1.0	22	23	24	25	26
2	2.0	12	12	13	13	14
3	5.0	5	5	6	6	7
		CANNIBALIZATION PERMITTED				
1	1.0	21	22	23	24	25
2	2.0	11	11	12	12	13
3	5.0	4	4	5	5	6

TABLE 4.4 Minimum Spare Allocations for a 10 Unit System with a Time Horizon of 10 Periods.

UNITS-N	10
TIME--T	10.00

COMP TYPE	MTBF	Minimum Probability of Spare Adequacy 0.70	0.75	0.80	0.85	0.90
		CANNIBALIZATION NOT PERMITTED				
1	10.00	12	12	13	13	14
2	50.00	3	3	3	3	4
3	100.00	1	2	2	2	2
		CANNIBALIZATION PERMITTED				
1	10.00	3	3	4	4	5
2	50.00	0	0	0	0	0
3	100.00	0	0	0	0	0

TABLE 4.5 Minimum Spare Allocations for a 10 Unit System with a Time Horizon of 100 Periods.

UNITS-N	10
TIME--T	100.00

COMP TYPE	MTBF	0.70	0.75	0.80	0.85	0.90
		MINIMUM PROBABILITY OF SPARE ADEQUACY				
CANNIBALIZATION NOT PERMITTED						
1	100.0	12	12	13	13	14
2	500.0	3	3	3	3	4
3	1000.0	1	2	2	2	2
CANNIBALIZATION PERMITTED						
1	100.0	3	3	4	4	5
2	500.0	0	0	0	0	0
3	1000.0	0	0	0	0	0

Constraint on Total Weight or Volume of Spares

If it is desired to analyze the operational cost model subject to a constraint on the total weight or volume of the spare inventory, the formulation would be similar to the classical "knapsack problem" of integer programming [47]. However, in the "knapsack problem" the value to the user is a linear function of the essentiality, or worth, of each item. This is not necessarily the case in the problem presented in this dissertation; the present worth of the revenue function is a nonlinear function of the number of units.

A constraint on the total weight of the spare inventory would be applicable in a situation such as modular sparing of processing units for a computer on-board an aircraft. Here, both the total weight and total volume may have restrictions applied to them. If both constraints are in effect, the required spare allocation would have to satisfy:

$$(4.13) \quad \sum_{j=1}^{P} w_j m_j \leq W_0, \text{ and}$$

$$\sum_{j=1}^{P} v_j m_j \leq V_0.$$

Here, w_j is the unit weight of the j^{th} component type, W_0 is the maximum weight allowed for all P component types, v_j is the volume of the j^{th} component type, and V_0 is the maximum allowable volume for all P component types.

Constraint on Total Investment in Spares

In some economic situations, there may be a budget limitation which restricts the available funds for purchasing the spare inventory components.

This constraint may be total, in that no more than L_o dollars may be spent for the entire spare inventory of $\sum_{j=1}^{P} m_j$ components, or it may be partial, in that a maximum amount may be spent only on "critical" spare components, with no funds allocated for other "less critical" component types. This restriction limits the spare inventory allocation to only those components which have available funding.

Under the first situation, in which the total inventory dollar value is limited to L_o dollars, the appropriate constraint is:

$$\sum_{j=1}^{P} a_j m_j \leq L_o. \tag{4.14}$$

Here, a_j is the cost of the j^{th} component type, excluding any discounts for large quantities purchased.

If there is the restriction that only certain spare types may be purchased with the limited total budget of L_o dollars, then the spare allocation scheme is more complex. Specifically, if only spare component type "q" may be purchased because of its importance, or its criticality for mission success, then the appropriate constraint is:

(4.15) $a_q m_q \leq L_o$.

This specifies a limit on the number of spares of component type q,

(4.16) $m_q \leq L_o/a_q$,

where a_q is the cost of a single component of type q.

If only two component types are permitted, say q and r, the appropriate constraint is:

(4.17) $a_q m_q + a_r m_r \leq L_o$,

with m_q, and m_r restricted to integers.

Constraint on Interval Availability

Since component reliability does not allow consideration for component repair or replacement actions, and system reliability is not an adequate indication of system dependability when repairs at the system level are permissible it is necessary to develop a meaningful constraint on a measure of system dependability which allows for the possibility of component repair. Interval Availability is such a measure.

Interval Availability was defined previously as the expected fractional amount of an interval of specified length that the system is in an operable state.

Given a specified time horizon, T, over which the system is expected to be operating, a constraint can be imposed on the cost model requiring that the system's (all N units) interval availability be in excess of α_0, for example. Thus it is required that the expected fraction of the time horizon, T, in which all N units are operable, be in excess of α_0.

However, in order to calculate the interval availability, one needs an accurate measure of the time dependent system availability, A(t). This may be difficult to formulate analytically for many system configurations. The interval availability, $A_I(T)$, is the average value of the system availability over a finite time horizon, T.

$$(4.18) \quad A_I(T) = (1/T) \int_0^T A(t)\,dt.$$

Thus, an appropriate constraint on interval availability is:

$$(4.19) \quad A_I(T) \geq \alpha_0 .$$

Maintainability Constraint

According to Rau [34], system maintainability is a characteristic of design and installation which is expressed as the probability that an item will be restored to specified conditions within a given period of time when maintenance actions are performed in accordance with prescribed procedures and resources.

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Under some circumstances, for example, the repair of emergency generating equipment within a hospital, there may be a constraint imposed on the maximum allowable downtime. One formulation of this maintainability constraint is:

$$(4.20) \qquad \Pr\{\text{downtime} \leq t_d\} \geq \alpha_0,$$

where α_0 is the minimum required probability of experiencing a downtime no greater than t_d.

System downtime is usually defined as the total time from system failure until return to service and includes the time to perform the replacement, or repair, as well as any administrative time; the administrative portion of the downtime is that required to procure spares and process any manuals or test equipment necessary for the repair or replacement action.

The major element of downtime which may be decreased by having an adequate supply of spares (in accordance with the criterion set forth in Equation (4.6)) is the portion of the administrative downtime which includes the lead time required when an additional spare procurement request is being processed.

However, if the administrative time is relatively small in comparison with the replacement time, then the system downtime can be approximated by the component replacement time. In the context of this research, the administrative portion of downtime will be assumed small relative to the replacement time, thus the total downtime and total replacement time are essentially

equivalent. Consequently, the chosen spare allocation does not significantly affect the expected downtime for a system under this formulation. Much more significant are the statistical parameters of the density function of the time to perform the replacements.

Summary and Discussion

This chapter outlined some specific constraints which should be considered during a system's design formulation. Several constraints were discussed, and a constraint on the probability of spare adequacy was analyzed in detail.

The constraint on spare adequacy was realistic in this application since the process of cannibalization is directly dependent upon, and in turn affects, the spare inventory level of each of the component types. The use of cannibalization, as illustrated by the tables, permitted a system configuration with fewer total spares for each component type.

Other constraints mentioned were a constraint on the total weight or volume of a specific spare allocation, a resource constraint on the total investment in spares, a requirement on a system's interval availability, and a maintainability constraint on system downtime. Each of these constraints may be applicable to a specific system under varying circumstances. The system user must decide initially, which if any, of the constraints, or combinations of constraints are applicable to his situation.

CHAPTER V

SOLUTION

Model Requirements

In order to effectively utilize this model, one needs specific cost and performance information on the system. It is necessary to know the component cost parameters; these may be obtained from prior purchase invoices or, on new systems, from vendor price estimates for a particular purchase quantity at a given time. One must also specify the number of units, N, which are to be fielded as a complete system, as well as estimates of the unit revenue function, V_i, i.e., the incremental revenue obtained when i units are in operation. If a system does not generate income, then a monetary measure of its operational worth to the user should be estimated. This estimate would then be used in the corresponding calculation of the present worth of the system's return or revenue function, K_4.

As far as the performance characteristics of the system are concerned, it is necessary to obtain estimates of the underlying failure and replacement density functions, $f_j(t)$, and $g_j(t)$, respectively, as well as their statistical parameters; the density function of the time to the first failure, $f_j^{(1)}(t)$ is obtained

from $f_j(t)$ in accordance with the procedure outlined in Appendix I. These functions could be obtained easily enough on currently fielded systems, by setting up an efficient data base collection system; however, for a new system such information is minimal, and one would have to use sound engineering judgment to estimate the system's performance characteristics. This is the major drawback in the use of this model on new systems.

Another requirement is that the net cost of alternative spare allocations must be compared over the same time horizon, T; this is because the cost model is a present worth technique.

As discussed in Chapter IV, there may be several constraints, or limitations, on the spare inventory allocation which greatly restrict the number of alternative allocations which must be examined. It is generally advisable to impose the constraint requirements first, in order to determine the feasible set of spare allocations, before performing any specific cost calculations. This will greatly reduce the total number of calculations and comparisons required. However, this method is only applicable if the active constraints limit the feasible spare allocations to a finite and comparatively small set. Otherwise, one would have to obtain an upper bound on the maximum number of spares for each component type, if possible, and perform a search over the feasible region. If it were not possible to obtain this upper bound, one would have to iterate the complete cost model across successive spare allocations and determine the trend in the cost calculations.

Nevertheless, this technique would only lead to a locally optimal solution, with no guarantee that the achieved solution was a global optimum.

A specific example problem will now be discussed in detail to illustrate the application of this particular cost model. There are two constraints which limit the feasible spare allocations to four possibilities. A constraint on the minimum spare adequacy probability determines a lower bound on the number of spares for each component type, while an investment constraint on the total dollar value of the spares provides an upper limit to the permissible number of spare allocations.

Example Problem

Consider the case of a small computer peripheral equipment leasing firm, Available Computer Enterprises (ACE). The firm operates a long term leasing service in which a support subsystem of computer peripherals is leased for an annual charge of $8000.00; the basic subsystem consists of a High Speed Line Printer, a Drum Plotter, and an Optical Card Reader.

ACE, being in the embrionic stage of its development in the computer leasing industry, has only two systems currently leased. Besides supplying the basic peripheral subsystems, ACE is responsible for all maintenance on the fielded systems, i.e. repairing or replacing failed peripheral components.

ACE has also included in the leasing contract a warranty provision stipulating that the customer's risk of experiencing a shutdown due to a critical spare shortage will be no greater than 10%, during the first year's service, i.e., both customers are assured that there is at least a 90% probability that there will be no critical spare shortage. Also stipulated in the contract is the provision that the customers are not obligated to pay for any time that the system is down for maintenance or repair. This requirement causes the revenue or return function, K_4, to be stochastic in nature, in that no revenue is generated when both systems are down.

In order to meet the constraint on spare shortages, ACE must stock enough backup spares; however, the firm is restricted in the number of spares it can stock, in that it only has a budget of $80,000.00 for backup spares.

Cannibalization has been suggested as a possible technique for decreasing the number of backup spares necessary to satisfy the spare sufficiency constraint, and meet the limitation on the maximum permissible investment in spares.

The solution of this allocation problem will consist of the ordered set (m_1^*, m_2^*, m_3^*), where m_1^* is the optimal number of spares for the Line Printer, m_2^* is the optimal number of spare Drum Plotters, and m_3^* is the optimal number of Optical Card Readers to be stocked. The values of the continuous annual interest rate to be used in present worth cost calculations are: 10%, 15%,

and 20% per year.

Table 5.1 contains the respective procurement, repair/replacement, and cannibalization cost coefficients for this system.

TABLE 5.1 Cost Coefficients ($)

	Procurement Cost	Repair/Replacement Cost	Cannibalization Cost
Line Printer	12000.00	330.00	1850.00
Drum Plotter	9050.00	250.00	2300.00
Optical Card Reader	5450.00	190.00	4000.00

Using data collected on other peripheral devices (based on the same technology), ACE officials were able to obtain realistic estimates of the failure and repair rates for the respective devices. Table 5.2 contains the failure and replacement rate data for each device, given on both an hourly and yearly basis, assuming an operating time of 8 hours per day, 6 days per week, 52 weeks per year; also included are the yearly MTBF and MTTR values, which were calculated under the assumption of exponential failure and replacement time.

TABLE 5.2 Component Operating Parameters for Available Computer Enterprises (ACE).

	Failure Rate		Replacement Rate		MTBF	MTTR
	($hr.^{-1}$)	($yr.^{-1}$)	($hr.^{-1}$)	($yr.^{-1}$)	(yr.)	(yr.)
Line Printer	5.0×10^{-4}	1.2480	0.00400	10.00	0.80128	0.10000
Drum Plotter	2.5×10^{-4}	0.6240	0.00333	8.32	1.60200	0.12019
Optical Card Reader	5.0×10^{-6}	0.0125	0.00333	8.32	80.12800	0.12019

Using the respective failure rates, the minimum sparing allocations required to satisfy the spare adequacy constraint of 90% for the first year's operation may be calculated; since the failure rates were much smaller than the replacement rates, it was permissible to use the Poisson model described in Chapter IV to calculate the required sparing allocations. Specifically, it was necessary to determine m_j for j = 1, 2, and 3, such that:

$$(5.1) \quad \prod_{j=1}^{3} \sum_{k=0}^{M_j} (2\lambda_j 1)^k \frac{\exp(-2\lambda_j 1)}{k!} \geq 0.90, \text{ where } M_j = 2+m_j-1.$$

The values given in Table 5.3 are the minimal spare allocations which must be checked against the spare adequacy constraint given in Equation (5.1); allocations are given for adequacy values of 70%, 75%, 80%, 85%, and 90%. For a 90% requirement, the minimum spare allocation, assuming cannibalization is permitted, is:

$$(5.2) \quad (m_1, m_2, m_3) = (4, 2, 0).$$

However, when the respective Poisson Sums are substituted into Equation (5.1), the product is 0.8978, which does not meet the 90% constraint. The spare adequacy constraint may be satisfied by stocking at least 4 Line Printers, 2 Drum Plotters, and 1 Optical Card Reader; this yields a spare sufficiency of 0.9209, at a cost of $71,550, which is $8,450 under the $80,000 budget constraint, quantitatively described as follows:

$$(5.3) \quad \sum_{j=1}^{P} a_j m_j \leq \$80,000.00.$$

TABLE 5.3 Minimum Required Spare Allocations for Available Computer Enterprises (ACE).

UNITS-N	2
TIME--T	1.00

COMP TYPE	MTBF	MINIMUM PROBABILITY OF SPARE ADEQUACY 0.70	0.75	0.80	0.85	0.90
CANNIBALIZATION NOT PERMITTED						
1	0.8012	3	3	4	4	5
2	1.6020	2	2	2	2	3
3	80.128	0	0	0	0	0
CANNIBALIZATION PERMITTED						
1	0.8012	2	2	3	3	4
2	1.6020	1	1	1	1	2
3	80.128	0	0	0	0	0

There are 4 feasible allocations which meet the spare adequacy constraint, yet do not violate the budget restriction: (4,2,1), (4,2,2), (4,3,0), and (5,2,0).

The cost model will be analyzed completely for each allocation, over time horizons of 5, 10, and 15 years.

The present worth of the procurement costs is not dependent upon the interest rate or time horizon, because it is incurred at time 0.

$$(5.4) \quad K_1 = 2 \sum_{j=1}^{3} a_j + \sum_{j=1}^{3} a_j m_j = \begin{cases} \$124{,}550.00 \text{ for } (4,2,1) \\ \$130{,}000.00 \text{ for } (4,2,2) \\ \$128{,}150.00 \text{ for } (4,3,0) \\ \$131{,}100.00 \text{ for } (5,2,0) \end{cases}$$

The present worth of the repair/replacement costs is calculated in accordance with Equation (3.12) (p. 29), and requires calculating the multiple convolution, $(f_j^{(1)}(t)*g_j(t))^{*i} * R_j^{(1)}(t)$, in order to obtain the probability of exactly i failures and i repair completions, for $i = 1, 2, \ldots, N+m_j-1$.

Under an exponential failure density, it is possible to convert the above multiple convolution into the same form as the marginal density functions of the time to the i^{th} failure of the j^{th} component type, $h_j^{(i)}(t)$. From Equation (3.20) (p. 34), $h_j^{(i)}(t) = f_j^{(1)}(t)^{*i} * g_j(t)^{*i-1}$. Since $R_j^{(1)}(t) = f_j^{(1)}(t)/\beta_j$, then

$$(5.5) \quad (f_j^{(1)}(t)*g_j(t))^{*i} * R_j^{(1)}(t) = f_j^{(1)}(t)^{*i+1} * g_j(t)^{*i}/\beta_j$$

$$= h_j^{(i+1)}(t)/\beta_j,$$

for $j = 1, 2, \ldots, P$, and $i = 1, 2, \ldots, N+m_j-1$.

Because of the complexity of the expressions for the multiple convolutions[4], it was necessary to perform the present worth integrations numerically, using the method of Gaussian Quadrature[5].

The calculation of K_3, the present worth of the expected cannibalization cost is similar to the calculation of K_2 except that, in addition, it is necessary to obtain products of double integrals, in order to calculate the time dependent cannibalization probability; these multiple integrals were calculated numerically, using the method of Gaussian Quadrature. The general form of the cannibalization probability was given in Equation (3.22) (p. 35).

After the time dependent cannibalization probability is calculated, one must multiply by the respective cannibalization cost coefficients, c_j, and the continuous compounding discount factor, exp(-rt), and then integrate over the required time horizon, T, again using Gaussian Quadrature.

For the 4 possible configurations, the present worth of the expected cannibalization cost was much smaller than the other costs. The reason for this is understood by examining the values of the cannibalization probability at the end of each time horizon considered. Table 5.4 contains the listings of the cannibalization

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[4] Appendix IV contains the analytic form of the multiple convolutions required for the repair/replacement, and cannibalization cost calculations.

[5] Appendix V outlines the method of Gaussian Quadrature which was used to numerically integrate the repair/replacement cost, cannibalization cost, and the revenue function.

TABLE 5.4 Cannibalization Probability as a Function of Time and Spare Configuration

T = 5 Years

(m_1, m_2, m_3)	$D_1(T)$	$D_2(T)$	$D_3(T)$
(4 2 1)	2.25×10^{-3}	2.06×10^{-3}	2.93×10^{-6}
(4 2 2)	9.54×10^{-5}	8.70×10^{-5}	1.43×10^{-9}
(4 3 0)	3.19×10^{-2}	9.42×10^{-3}	3.30×10^{-3}
(5 2 0)	1.58×10^{-2}	5.51×10^{-2}	3.47×10^{-3}

T = 10 Years

(m_1, m_2, m_3)	$D_1(T)$	$D_2(T)$	$D_3(T)$
(4 2 1)	1.23×10^{-2}	1.07×10^{-2}	4.33×10^{-6}
(4 2 2)	9.99×10^{-4}	8.77×10^{-4}	2.91×10^{-9}
(4 3 0)	1.17×10^{-1}	3.62×10^{-2}	4.86×10^{-3}
(5 2 0)	6.03×10^{-2}	1.65×10^{-1}	4.39×10^{-3}

T = 15 Years

(m_1, m_2, m_3)	$D_1(T)$	$D_2(T)$	$D_3(T)$
(4 2 1)	2.72×10^{-2}	2.43×10^{-2}	4.40×10^{-6}
(4 2 2)	3.23×10^{-3}	2.91×10^{-3}	2.96×10^{-9}
(4 3 0)	1.83×10^{-1}	6.16×10^{-2}	4.91×10^{-3}
(5 2 0)	9.76×10^{-2}	2.65×10^{-1}	4.43×10^{-3}

probability, $D_j(T)$, for each spare configuration over the three time horizons considered[6].

In order to obtain the present worth of the return or revenue function, K_4, one must analyze all possible state transitions between the "up" and "down" states for each of the three component types. The analysis must include all possible transitions for each spare allocation. For example, component type 1 must be analyzed assuming 4 or 5 spares, component type 2 can have 2 or 3 spares, and component type 3 can have 0, 1, or 2 spares.

Because of the assumption of statistical independence of the respective failure and replacement times, stated in Chapter I, one can obtain the unit availability function, $P_i(t)$, by calculating the respective component availability functions, i.e. the probability of 0, 1, or 2 components of each type being up at any time, t.

Define $q_{ij}(t)$ as the probability of having i components of the j^{th} type being up at any time t, for i=0, 1, or 2, and j=1, 2, and 3. $P_i(t)$ was defined earlier as the time dependent probability of having exactly i units up at time t. Then,

$$(5.6)\quad P_2(t) = \Pr\{\text{exactly 2 units are up at time t.}\}$$

$$= \Pr\left\{\begin{array}{l}\text{2 components of type 1 are up, 2 components}\\ \text{of type 2 are up, and 2 of type 3 are}\\ \text{up at time t.}\end{array}\right\}$$

$$= q_{21}(t)\, q_{22}(t)\, q_{23}(t).$$

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[6]For each time horizon, T, ten values of the cannibalization probability were calculated; the last being T x R_{10}, where R_{10}(=0.9869) was the 10^{th} root used for the Gaussian Quadrature integration.

Similarly[7],

(5.7) $P_0(t) = \Pr$ {exactly 0 units are up at time t.}

$$= \Pr \left\{ \begin{array}{l} \text{0 components of type 1 are up, } \underline{\text{or}} \\ \text{0 components of type 2 are up, } \underline{\text{or}} \\ \text{0 components of type 3 are up at time t.} \end{array} \right\}$$

$$\begin{aligned} &= q_{01}q_{02}q_{03} + q_{01}q_{02}q_{13} + q_{01}q_{12}q_{03} + q_{01}q_{12}q_{13} \\ &+ q_{11}q_{02}q_{03} + q_{11}q_{02}q_{13} + q_{11}q_{12}q_{03} + q_{01}q_{02}q_{23} \\ &+ q_{01}q_{22}q_{03} + q_{01}q_{22}q_{23} + q_{21}q_{01}q_{03} + q_{21}q_{02}q_{23} \\ &+ q_{21}q_{22}q_{03} + q_{01}q_{12}q_{23} + q_{01}q_{21}q_{13} + q_{11}q_{22}q_{03} \\ &+ q_{21}q_{12}q_{03} + q_{11}q_{02}q_{23} + q_{21}q_{02}q_{13} . \end{aligned}$$

Since there must be 0, 1, or 2 units up at any time t, then,

(5.8) $P_1(t) = 1 - P_0(t) - P_2(t)$.

In order to calculate each of the q_{ij} terms, a state transition analysis is performed by modelling the transitions of spare consumptions as a Markov Process, with an absorbing state; the absorbing state is the state achieved after the $(N+m_j-1)^{th}$ replacement of the j^{th} component type, i.e. after (N-1) components have been cannibalized.

[7]For notational convenience, the time dependence of $q_{ij}(t)$ will not be shown in this, and any subsequent calculations.

State Transition Analysis

For any given component type, define $S_{i,k}(t)$ as the time dependent probability of being in the state (i,k); i.e. having i components up with k spares remaining. The state transition diagrams for each possible spare configuration are contained in Figures 5.1 through 5.5. The possibility of using a cannibalized component to replace a failed component is illustrated by the final transition to state (1,-1). The negative index for the number of spares remaining indicates that the replacement came from a cannibalized component.

The last possible transition, to state (1,-1), is the transition to a cannibalized state. After cannibalization, one can not achieve full system operational capability because the very process of cannibalization eliminates one unit from active service; consequently, the best capability one achieves after cannibalization is a working system of 1 operational unit.

Figure 5.1 contains the state transition diagram for component type 1, having 5 spares initially. There are 19 possible states; the state transition equations will be transformed into a system of 19 coupled first order differential equations.

Referring to Figure 5.1, the following analysis results: During an arbitrary interval of duration dt, component type 1 will go from a state in which 2 units are up with 5 spares remaining, state (2,5), to a state with 1 unit up and 5 spares

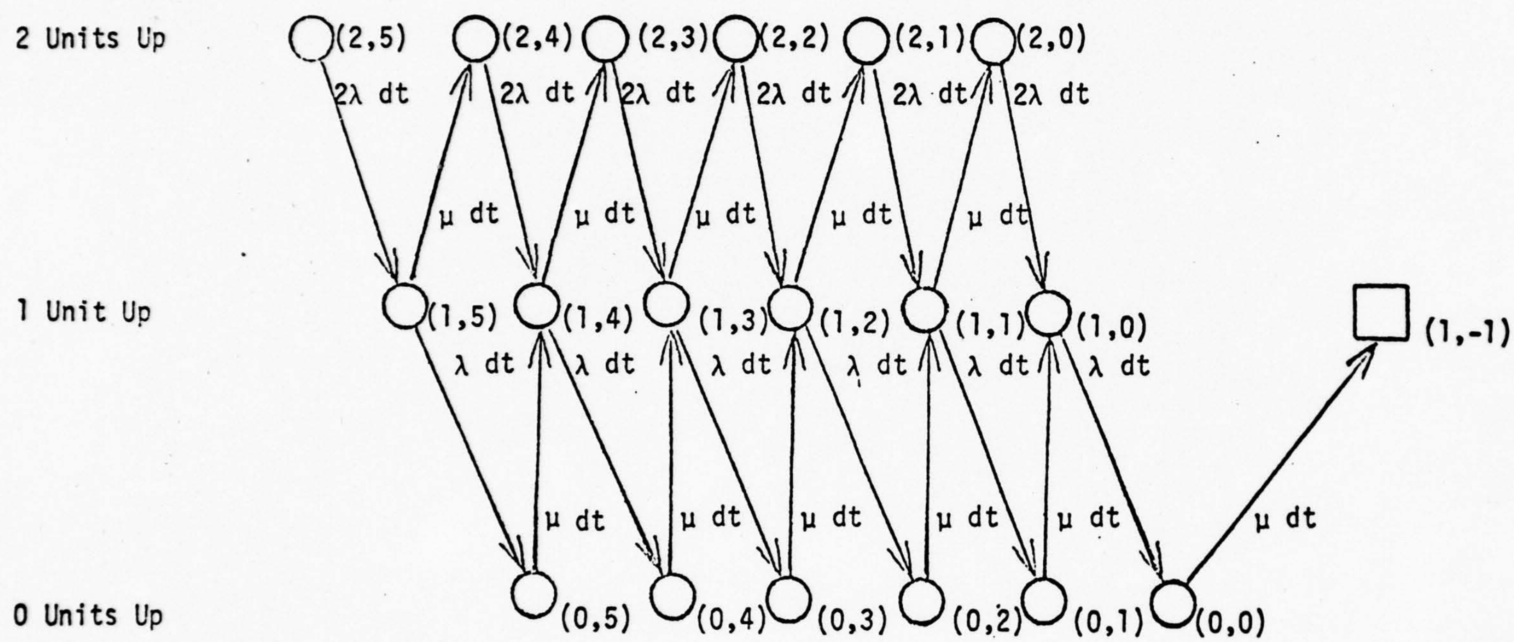

Figure 5.1 State Transition Diagram for a Configuration with 5 Spares Initially Provisioned.

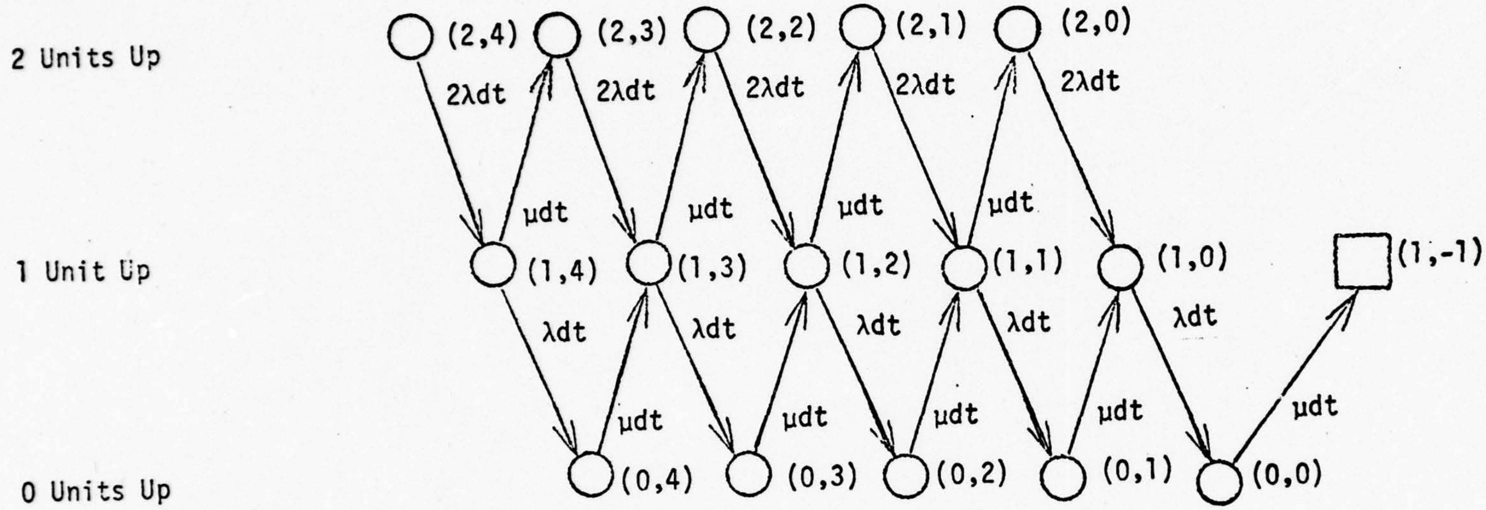

Figure 5.2 State Transition Diagram for a Configuration with 4 Spares Initially Provisioned.

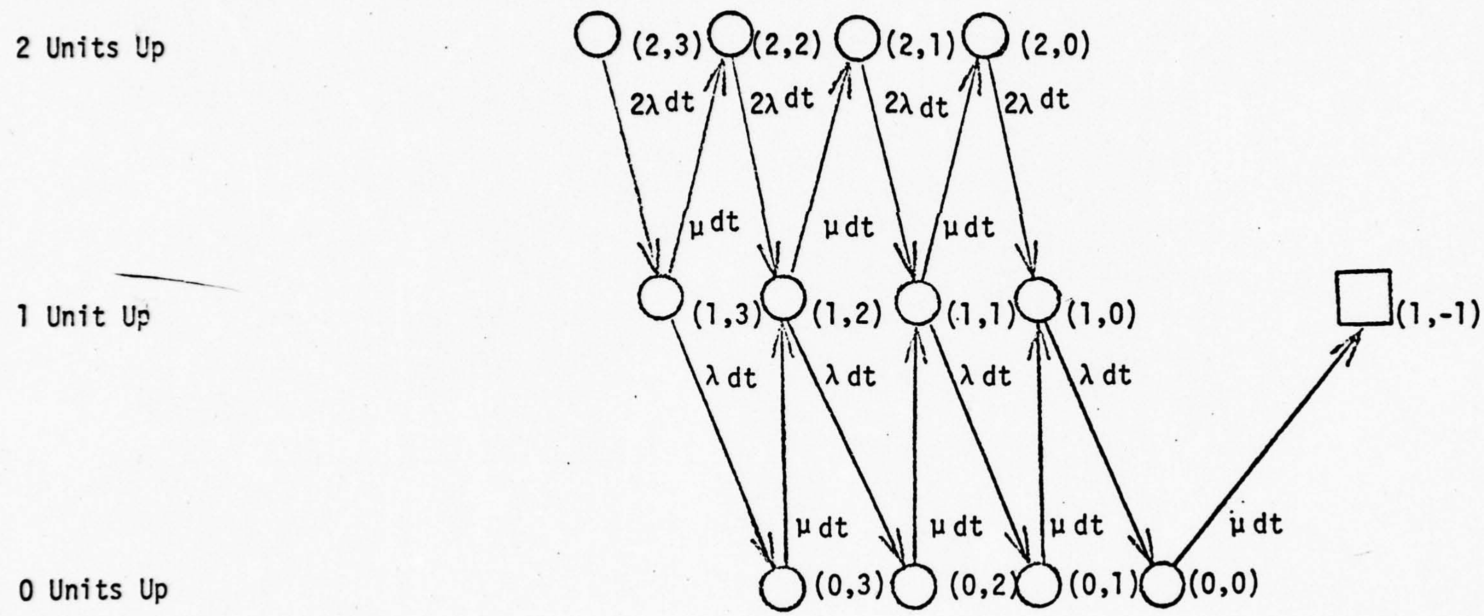

Figure 5.3 State Transition Diagram for a Configuration with 3 Spares Initially Provisioned.

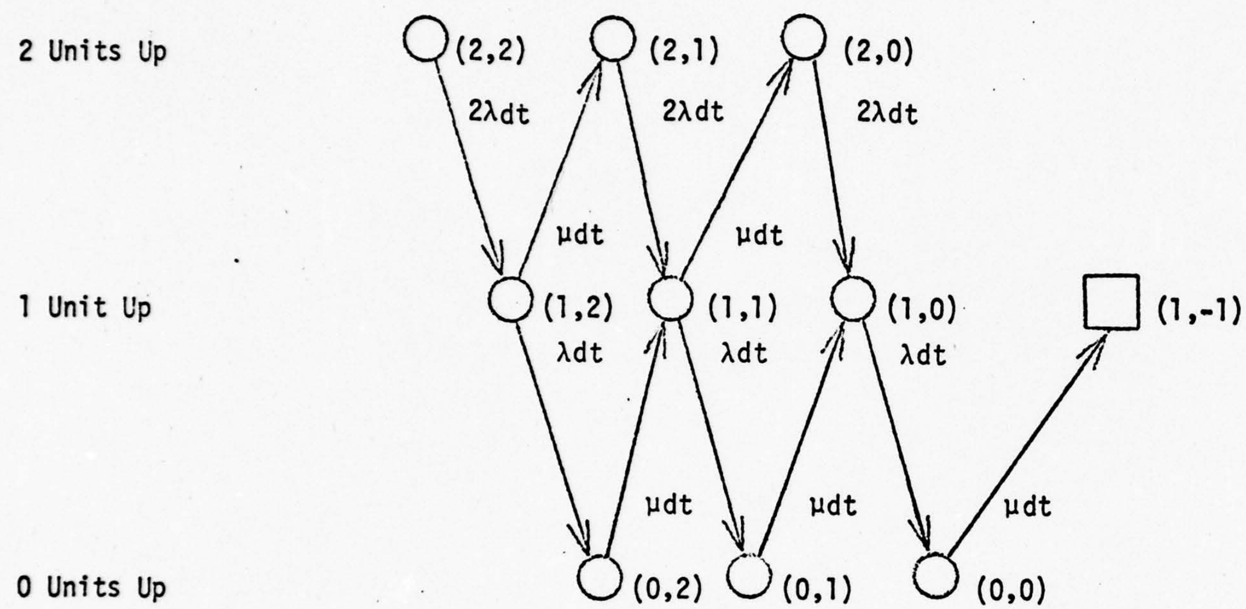

Figure 5.4 State Transition Diagram for a Configuration with 2 Spares Initially Provisioned.

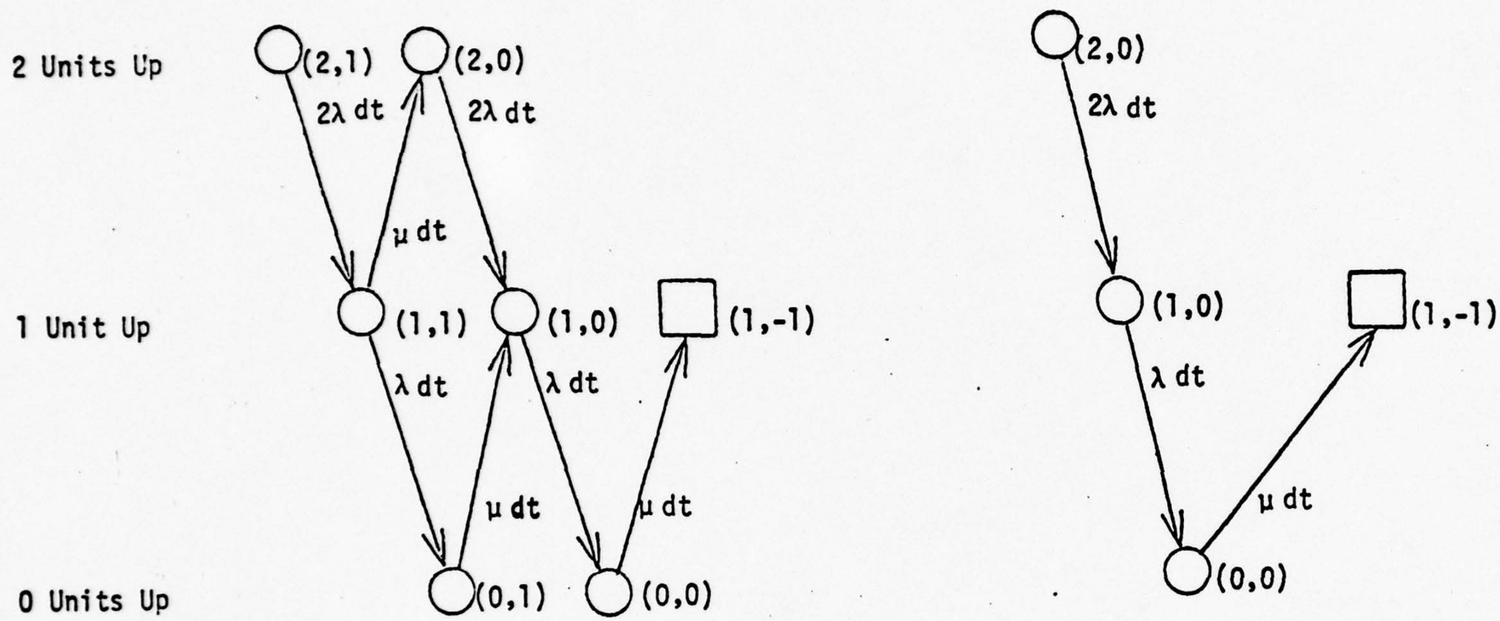

Figure 5.5 State Transition Diagram for Configurations having 1 Spare, and 0 Spares Initially Provisioned.

remaining, state (1,5), with probability $2\lambda_1 dt$. Thus the probability that it remains in state (2,5) is $(1 - 2\lambda_1 dt)$. Once it is in state (1,5), it can remain in that state with probability $(1 - \mu_1 dt - \lambda_1 dt)$, or it can undergo a replacement and go to state (2,4) with probability $\mu_1 dt$, or it can fail with probability $\lambda_1 dt$, and go to state (0,5). The 19 state transition equations for component type 1 follow.

$$(5.9)\qquad S_{2,5}(t+dt) = (1-2\lambda_1 dt)\, S_{2,5}(t) \qquad\qquad S_{2,5}(0) = 1.0$$

$$S_{1,5}(t+dt) = (1-\mu_1 dt-\lambda_1 dt)\, S_{1,5}(t) + 2\lambda_1 dt\, S_{2,5}(t)$$

$$S_{0,5}(t+dt) = (1-\mu_1 dt)\, S_{0,5}(t) + \lambda_1 dt\, S_{1,5}(t)$$

$$S_{2,4}(t+dt) = (1-2\lambda_1 dt)\, S_{2,4}(t) + \mu_1 dt\, S_{1,5}(t)$$

$$\begin{aligned} S_{1,4}(t+dt) = {} & (1\ \lambda_1 dt-\mu_1 dt)\, S_{1,4}(t) + \mu_1 dt\, S_{0,5}(t) \\ & + 2\lambda_1 dt\, S_{2,4}(t) \end{aligned}$$

$$S_{0,4}(t+dt) = (1-\mu_1 dt)\, S_{0,4}(t) + \lambda_1 dt\, S_{1,4}(t)$$

$$S_{2,3}(t+dt) = (1-2\lambda_1 dt)\, S_{2,3}(t) + \mu_1 dt\, S_{1,4}(t)$$

$$\begin{aligned} S_{1,3}(t+dt) = {} & (1-\lambda_1 dt-\mu_1 dt)\, S_{1,3}(t) + \mu_1 dt\, S_{0,4}(t) \\ & + 2\lambda_1 dt\, S_{2,3}(t) \end{aligned}$$

$$S_{0,3}(t+dt) = (1-\mu_1 dt)\, S_{0,3}(t) + \lambda_1 dt\, S_{1,3}(t)$$

(5.9) $S_{2,2}(t+dt) = (1-2\lambda_1 dt)\, S_{2,2}(t) + \mu_1 dt\, S_{1,3}(t)$

(cont'd)

$$S_{1,2}(t+dt) = (1-\lambda_1 dt-\mu_1 dt)\, S_{1,2}(t) + \mu_1 dt\, S_{0,3}(t) + 2\lambda_1 dt\, S_{2,2}(t)$$

$$S_{0,2}(t+dt) = (1-\mu_1 dt)\, S_{0,2}(t) + \lambda_1 dt\, S_{1,2}(t)$$

$$S_{2,1}(t+dt) = (1-2\lambda_1 dt)\, S_{2,1}(t) + \mu_1 dt\, S_{1,2}(t)$$

$$S_{1,1}(t+dt) = (1-\mu_1 dt-\lambda_1 dt)\, S_{1,1}(t) + \mu_1 dt\, S_{0,2}(t) + 2\lambda_1 dt\, S_{2,1}(t)$$

$$S_{0,1}(t+dt) = (1-\mu_1 dt)\, S_{0,1}(t) + \lambda_1 dt\, S_{1,1}(t)$$

$$S_{2,0}(t+dt) = (1-2\lambda_1 dt)\, S_{2,0}(t) + \mu_1 dt\, S_{1,1}(t)$$

$$S_{1,0}(t+dt) = (1-\lambda_1 dt)\, S_{1,0}(t) + \mu_1 dt\, S_{0,1}(t) + 2\lambda_1 dt\, S_{2,0}(t)$$

$$S_{0,0}(t+dt) = (1-\mu_1 dt)\, S_{0,0}(t) + \lambda_1 dt\, S_{1,0}(t)$$

$$S_{1,-1}(t+dt) = \mu_1 dt\, S_{0,0}(t)$$

Using the definition of the derivative,

$$(5.10)\quad \dot{S}_{i,k}(t) = \lim_{dt \to 0} \left\{ \frac{S_{i,k}(t+dt) - S_{i,k}(t)}{dt} \right\},$$

the following system of first order differential equations may be constructed for the first component type when it has 5 spares.

$$(5.11)\quad \dot{S}_{2,5}(t) = -2\,\lambda_1 S_{2,5}(t) \qquad\qquad S_{2,5}(0) = 1.0$$

$$\dot{S}_{1,5}(t) = -(\lambda_1 + \mu_1)\,S_{1,5}(t) + 2\,\lambda_1 S_{2,5}(t)$$

$$\dot{S}_{0,5}(t) = -\mu_1\,S_{0,5}(t) + \lambda_1 S_{1,5}(t)$$

$$\dot{S}_{2,4}(t) = -2\,\lambda_1 S_{2,4}(t) + \mu_1 S_{1,5}(t)$$

$$\dot{S}_{1,4}(t) = -(\lambda_1 + \mu_1)\,S_{1,4}(t) + \mu_1\,S_{0,5}(t) + 2\,\lambda_1 S_{2,4}(t)$$

$$\dot{S}_{0,4}(t) = -\mu_1\,S_{0,4}(t) + \lambda_1\,S_{1,4}(t)$$

$$\dot{S}_{2,3}(t) = -2\lambda_1 S_{2,3}(t) + \mu_1 S_{1,4}(t)$$

$$\dot{S}_{1,3}(t) = -(\lambda_1 + \mu_1)\,S_{1,3}(t) + \mu_1\,S_{0,4}(t) + 2\lambda_1\,S_{2,3}(t)$$

$$\dot{S}_{0,3}(t) = -\mu_1\,S_{0,3}(t) + \lambda_1\,S_{1,3}(t)$$

$$\dot{S}_{2,2}(t) = -2\lambda_1\,S_{2,2}(t) + \mu_1\,S_{1,3}(t)$$

$$\dot{S}_{1,2}(t) = -(\lambda_1 + \mu_1)\,S_{1,2}(t) + \mu_1 S_{0,3}(t) + 2\lambda_1\,S_{2,2}(t)$$

(5.11) (cont'd)

$$\dot{S}_{0,2}(t) = -\mu_1 S_{0,2}(t) + \lambda_1 S_{1,2}(t)$$

$$\dot{S}_{2,1}(t) = -2\lambda_1 S_{2,1}(t) + \mu_1 S_{1,2}(t)$$

$$\dot{S}_{1,1}(t) = -(\lambda_1 + \mu_1) S_{1,1}(t) + \mu_1 S_{0,2}(t) + 2\lambda_1 S_{2,1}(t)$$

$$\dot{S}_{0,1}(t) = -\mu_1 S_{0,1}(t) + \lambda_1 S_{1,1}(t)$$

$$\dot{S}_{2,0}(t) = -2\lambda_1 S_{2,0}(t) + \mu_1 S_{1,1}(t)$$

$$\dot{S}_{1,0}(t) = -\lambda_1 S_{1,0}(t) + \mu_1 S_{0,1}(t) + 2\lambda_1 S_{2,0}(t)$$

$$\dot{S}_{0,0}(t) = -\mu_1 S_{0,0}(t) + \lambda_1 S_{1,0}(t)$$

$$\dot{S}_{1,-1}(t) = \mu_1 S_{0,0}(t)$$

From the solution of these differential equations[8], it is possible to obtain the time dependent probability of 0, 1, or 2 components of the first type being "up", or available, at any given time, q_{01}, q_{11}, or q_{21}.

(5.12) $$q_{01} = S_{0,5}(t) + S_{0,4}(t) + S_{0,3}(t) + S_{0,2}(t) + S_{0,1}(t) + S_{0,0}(t).$$

[8]Appendix VI contains the differential equations for the other configurations, i.e. a 2 unit system with 4, 3, 2, 1, or 0 spares for any component type.

$$(5.13)\quad q_{11} = S_{1,5}(t) + S_{1,4}(t) + S_{1,3}(t) + S_{1,2}(t) + S_{1,1}(t) + S_{1,0}(t) + S_{1,-1}(t)$$

$$(5.14)\quad q_{21} = S_{2,5}(t) + S_{2,4}(t) + S_{2,3}(t) + S_{2,2}(t) + S_{2,1}(t) + S_{2,0}(t)$$

Once the time dependent unit availability function, $P_i(t)$, is determined, the present worth of the return or revenue function is calculated using Gaussian Quadrature to perform the following integration:

$$(5.15)\quad K_4 = \int_0^T \sum_{i=1}^{2} V_i\, P_i(t) \exp(-rt)\, dt,$$

for r = 10%, 15%, and 20%; T = 5, 10, and 15 years; V_1 = \$8000.00 per year, V_2 = \$16000.00 per year.

Tables 5.5, 5.6, and 5.7 contain the final solutions for the example problem for each of the three time horizons, and with continuous interest rates of 10%, 15%, and 20% per year, respectively. In each case, the allocation of 4 Line Printers, 2 Drum Plotters, and 1 Card Reader resulted in the minimum net present worth of the total costs, K_0. This is primarily due to the overwhelming effect of the respective procurement costs, K_1, in comparison with the other costs. In each case, allocation (4,2,1) had the minimum procurement cost and replacement cost, while allocation (4,2,2) had the smallest cannibalization cost. Allocation (4,3,0) yielded the maximum revenue but it was not large

TABLE 5.5 Present Worth of the Total Costs for each Spare Configuration, with a 10% Annual Interest Rate.

K_0 = Net Present Worth of Total Operational Costs ($).

T = 5 Years (m_1,m_2,m_3)	K_0	=	K_1	+	K_2	+	K_3	-	K_4
(4 2 1)	80843.72		124550.00		2451.68		6.24		46164.20
(4 3 0)	81716.23		128150.00		2826.15		68.63		49328.55
(4 2 2)	86275.25		130000.00		2451.79		0.23		46176.77
(5 2 0)	94466.79		131100.00		2979.33		137.71		39750.25

T = 10 Years (m_1,m_2,m_3)	K_0	=	K_1	+	K_2	+	K_3	-	K_4
(4 2 1)	61873.78		124550.00		2619.05		67.55		65362.82
(4 3 0)	62965.42		128150.00		3121.49		563.24		68869.31
(4 2 2)	67246.08		130000.00		2620.25		4.28		65378.45
(5 2 0)	76501.27		131100.00		3171.47		945.56		58715.76

T = 15 Years (m_1,m_2,m_3)	K_0	=	K_1	+	K_2	+	K_3	-	K_4
(4 2 1)	50505.70		124550.00		2698.58		175.62		76918.50
(4 3 0)	52107.65		128150.00		3184.96		1154.69		80382.00
(4 2 2)	55784.68		130000.00		2702.56		15.37		76933.25
(5 2 0)	65924.56		131100.00		3236.74		1894.70		70306.88

TABLE 5.6 Present Worth of the Total Costs for each Spare Configuration, with a 15% Annual Interest Rate.

K_0 = Net Present Worth of Total Operational Costs ($).

T = 5 Years (m_1,m_2,m_3)	K_0	= K_1	+ K_2	+ K_3	- K_4
(4 2 1)	84883.72	124550.00	2197.64	5.06	41868.98
(4 3 0)	86104.14	128150.00	2514.47	55.71	44616.04
(4 2 2)	90318.05	130000.00	2197.73	0.18	41879.86
(5 2 0)	97772.14	131100.00	2649.47	112.16	36089.49

T = 10 Years (m_1,m_2,m_3)	K_0	= K_1	+ K_2	+ K_3	- K_4
(4 2 1)	71671.83	124550.00	2317.32	46.41	55241.91
(4 3 0)	73032.29	128150.00	2728.46	392.37	58238.54
(4 2 2)	77065.88	130000.00	2318.14	2.88	55255.14
(5 2 0)	85271.24	131100.00	2788.60	663.02	49280.38

T = 15 Years (m_1,m_2,m_3)	K_0	= K_1	+ K_2	+ K_3	- K_4
(4 2 1)	65513.94	124550.00	2359.16	104.06	61499.28
(4 3 0)	67158.43	128150.00	2761.93	710.16	64463.66
(4 2 2)	70858.67	130000.00	2361.46	8.75	61511.54
(5 2 0)	79560.11	131100.00	2825.07	1172.81	55537.77

TABLE 5.7 Present Worth of the Total Costs for each Spare Configuration, with a 20% Annual Interest Rate.

K_0 = Net Present Worth of Total Operational Costs ($).

T = 5 Years (m_1,m_2,m_3)	K_0	=	K_1	+	K_2	+	K_3	-	K_4
(4 2 1)	88377.99		124550.00		1975.37		4.11		38151.49
(4 3 0)	89897.16		128150.00		2244.02		45.27		40542.13
(4 2 2)	93814.66		130000.00		1975.44		0.15		38160.93
(5 2 0)	100630.46		131100.00		2362.97		91.46		32923.97

T = 10 Years (m_1,m_2,m_3)	K_0	=	K_1	+	K_2	+	K_3	-	K_4
(4 2 1)	79130.12		124550.00		2061.31		32.14		47513.33
(4 3 0)	80739.18		128150.00		2399.60		275.54		50085.96
(4 2 2)	84539.28		130000.00		2061.87		1.96		47524.55
(5 2 0)	91890.06		131100.00		2464.05		468.94		42142.93

T = 15 Years (m_1,m_2,m_3)	K_0	=	K_1	+	K_2	+	K_3	-	K_4
(4 2 1)	75780.88		124550.00		2082.81		63.04		50914.97
(4 3 0)	77580.22		128150.00		2416.77		477.18		53463.73
(4 2 2)	81163.89		130000.00		2084.16		5.08		50925.35
(5 2 0)	88804.84		131100.00		2484.90		744.15		45524.21

enough, in comparison with the other costs, to alter the optimal spare configuration.

To see the effect of the stochastic nature of the revenue or return function, one needs only to calculate upper and lower bounds on the expected revenue by assuming $P_i(t) = 1$, for all time. Then, a lower bound on the revenue function can be calculated by assuming 1 unit was operational throughout the entire time horizon yielding an annual revenue of \$8000.00 per year; likewise, an upper bound can be calculated by assuming both units were operational throughout the entire time duration. With the relatively small MTTR values, in comparison with the respective MTBF values, one would expect the present worth of the revenue function to lie between the bounds K_4^L, and K_4^U, where the respective bounds are calculated as follows:

$$(5.16)\quad K_4^L = \int_0^T V_1 \exp(-rt)\,dt = V_1 (1 - \exp(-rT))/r = \$8000 (1 - \exp(-rT))/r.$$

$$(5.17)\quad K_4^U = \int_0^T V_2 \exp(-rt)\,dt = V_2 (1 - \exp(-rT))/r = \$16000 (1 - \exp(-rT))/r.$$

Table 5.8 contains values of the upper and lower bounds on the present worth of the expected revenue function for interest rates of 10%, 15%, and 20% per year, over time horizons of 5, 10,

TABLE 5.8 Bounds on the Expected Revenue Function.

INTEREST RATE (r)	10%		15%		20%	
TIME (T)	K_4^L	K_4^U	K_4^L	K_4^U	K_4^L	K_4^U
5	31477.52	62955.04	28140.43	56280.86	25284.80	50569.63
10	50569.63	101139.26	41433.05	82866.10	34586.58	69173.17
15	62149.57	124299.15	47712.04	95424.08	38008.52	76017.03

and 15 years. As expected, the calculated values of the revenue function, K_4, lie between the upper and lower bounds, for all possible configurations.

Summary and Discussion

This chapter outlined the necessary requirements for utilizing this cannibalization cost model to determine spare provisioning requirements. It is a present worth analysis, so the derived cost comparisons are only valid if compared over equal time horizons.

An example problem was presented in which there were two active constraints: a performance constraint, which specified the minimum required probability of spare adequacy, and a resource constraint, which imposed a limitation on the total investment in spares. The complexity of the cost calculations was formidable for all but the smallest system configurations; this is mainly due to the complexity of the expression for the cannibalization probability, products of (P-1) double integrals for each of the P component types. The problem is further complicated by the necessity of calculating the multiple convolutions for the marginal density functions of the terminal failure times.

Gaussian Quadrature proved to be an efficient numerical technique for performing both the multiple integrations for the time dependent cannibalization probability, and the single integrations for each of the respective present worth calculations.

CHAPTER VI

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

The primary objective of this research was to develop an operational cost model which allowed one to analyze the effects of cannibalization on spare inventory allocations. The model was not restricted to areas in which instantaneous replenishment was assumed; it was developed under the assumption of exponential failures and a general replacement density function.

A mathematical formulation of the model is restated below: Minimize $K_0 = K_1 + K_2 + K_3 - K_4$, where K_0 is the present worth of the net operational costs, K_1 is the present worth of the manufacturing or procurement costs, K_2 is the present worth of the repair/replacement costs, K_3 is the present worth of the cannibalization costs, and K_4 is the present worth of the return or revenue function. The equations for each of the major terms are defined as follows:

(6.1) $$K_1 = N \sum_{j=1}^{P} a_j + \sum_{j=1}^{P} a_j m_j .$$

(6.2) $$K_2 = \int_0^T \sum_{j=1}^{P} b_j(t) \cdot \sum_{i=1}^{N+m_j-1} i(f_j^{(1)} * g_j(t))^{*i} * R_j^{(1)}(t) \exp(-rt)dt.$$

(6.3) $$K_3 = \int_0^T (N-1) \sum_{j=1}^{P} c_j D_j(t) \exp(-rt)dt,$$ where

$D_j(t)$ is the time dependent cannibalization probability, defined as follows:

$$(6.4) \quad D_j(t) = \prod_{\substack{k=1 \\ k \neq j}}^{P} \int_o^t \int_o^y h_j^{(N+m_j-1)}(x)\, h_k^{(N+m_k-1)}(y)\, dx\, dy, \text{ where}$$

$h_j^{(N+m_j-1)}(t)$ is the marginal density function of the time to the $(N+m_j-1)^{th}$ failure of the j^{th} component type; the analogous definition holds for $h_k^{(N+m_k-1)}(t)$. The revenue function is expressed as follows:

$$(6.5) \quad K_4 = \int_o^T \sum_{i=o}^{N} V_i \, P_i(t) \exp(-rt)dt.$$

The general model is also applicable when there are one or more constraints, which usually provide bounds on the permissible spare allocations. A constraint on the minimum required spare adequacy results in a lower bound on the number of spares, while an investment constraint usually provides an upper bound. Other applicable constraints include a constraint on the total weight or volume of a specific spare allocation, a requirement on the system's interval availability, and a maintainability constraint on system downtime.

A specific example problem was solved which illustrated the applicable cost calculations. Gaussian Quadrature was used to perform the required integrations.

Because of the complexity of the marginal density functions of the terminal failure times for large system configurations, a general allocation model was not computationally feasible; however, one

should be able to model his allocation problem using the methodology developed, and proceed to a complete solution using the same steps presented in the example problem.

The methodology developed in this research can be put to use in either of two contexts. First, it provides a tool, based on equipment reliability and maintainability data, for an accurate prediction of net operational costs on a present worth basis. Second, the methodology provides a mechanism for showing the relationship between a spare stockage level and the corresponding risk of a critical spare shortage. Consequently, this analysis makes it possible to study the interaction between net costs and the possible shortage risks over the system's lifetime.

The application of this cost model can benefit both industrial and military procurement strategies. The analysis is especially useful when advancing technology hastens the process of obsolescence of current equipment. Management can utilize this model to determine the best of several alternative spare allocations, based upon the present worth of the net operational costs, assuming the interest rate and time horizon are known. Consequently, short run decisions can be made regarding the effectiveness of purchasing a new system, versus keeping the old. The present worth analysis allows management to make such a decision if the cost comparison is made over the same time interval.

The model is also useful in situations having stringent requirements on spare adequacy, i.e. α_0 very close to 1.0. Spare

provisioning for military weapon systems falls into such a category.

As in most research endeavors, there is always room for additional work; this research is no exception. Some further extensions of this model are outlined below.

There are many situations in which the continuous annual interest rate is not deterministic, i.e. it may be a random variable over a specific time horizon. This is especially true during a volatile money market, when the Federal Reserve Board may, on a relatively short notice, increase the going interest rate charged its member banks. Such an action has a "domino" effect in the financial world, with banks subsequently raising the prime rate charged their most credit-worthy customers. Consequently, a stochastic analysis of the present worth of the net operational costs under cannibalization, which considers the random nature of the interest rate, would be a valuable contribution to the state of the art. One may be able to determine a probability density function of the present worth of the net operational costs, and develop risk profiles which illustrate the probability of the present worth exceeding a specified level. This analysis would be extremely beneficial if applied prior to the procurement of a complete system or spare configuration.

Another extension of this model would be to consider the effects of taxes and inflation on the optimal spare allocation, under the same assumptions used herein. With this analysis, the model could be tailored to the specific industries which are subject to known

tax rates. Inflation is another aspect. Without considering inflation, a company may experience a cash shortfall equal to the amount by which an equipment's accumulated depreciation fails to cover its replacement cost. This possibility should be considered when procuring a new system or its spare configuration; a model which includes the inflationary effects on procurement and replacement costs would be valuable.

A further extension of this basic model is a consideration of catastrophic failures, i.e. where the failure of one or more components causes the failure of the entire unit. This was not considered in the analysis contained herein, because of the assumption of statistically independent failure times. A catastrophic failure analysis is applicable in hostile military environments, where enemy actions directed against a single component or subsystem destroy the complete unit. Consequently, cannibalization of that unit is prevented.

Continuing in the same area, one may be able to develop a cannibalization cost model when all the components are subject to multiple types of failure or when a unit is permitted to operate under a degraded level of performance. Such an analysis necessitates a modified failure density function for each component type.

The present worth analysis of cannibalization in determining spare provisioning requirements has been an enlightening area of research. The author hopes that this work will help motivate others to continue in the same area.

APPENDIX I

CALCULATION OF THE DENSITY FUNCTION OF THE ORDERED FAILURE TIMES

The term $f_j^{(i)}(t)$ is the density function of the i^{th} ordered failure time of the j^{th} component type; it is obtained using the methods of order statistics. In general, the probability density function of the q^{th} order statistic from a random sample of size L, from a continuous population, x, $-\infty < x < \infty$ is:

$$\text{(A1.1)} \quad p^{(q)}(x) = \binom{L}{q-1,\ 1,\ L-q} (P(x))^{q-1}\ p(x)\ (1-P(x))^{L-q}$$

$$= \frac{L!}{(q-1)!\ 1!\ (L-q)!}\ (P(x))^{q-1}\ p(x)\ (1-P(x))^{L-q}\ ,$$

where p(x) is the continuous probability density function of the random variable X, and P(x) is the corresponding cumulative distribution function:

$$\text{(A1.2)} \quad P(x) = \Pr\ \{X \leq x\}.$$

For the general system configuration of Figure 3.1 (p. 21), $f_j^{(i)}(t)$ is calculated below:

$$\text{(A1.3)}\quad f_j^{(i)}(t) = \binom{N}{i-1,\ 1,\ N-i} (F_j(t))^{i-1}\ f_j(t)\ (1-F_j(t))^{N-i},$$

for $i = 1, 2, \ldots, N$, and

$$f_j^{(i)}(t) = \binom{N+m_j}{i-1,\ 1,\ N+m_j-i} (F_j(t))^{i-1}\ f_j(t)\ (1-F_j(t))^{N+m_j-i},$$

for[1] $i = N+1, N+2, \ldots, N+m_j-1$.

The term $f_j(t)$ is the probability density function of the unordered time to failure of the j^{th} component type; $F_j(t)$ is the cumulative probability distribution function of the unordered time to failure of the j^{th} component type.

Using Equation (A1.3), the expression for $f_j^{(1)}(t)$, as well as $R_j^{(1)}(t)$, is derived as follows:

$$\text{(A1.4)}\quad f_j^{(1)}(t) = \binom{N}{0,\ 1,\ N-1} F_j(t)^0\ f_j(t)\ (1-F_j(t))^{N-1}$$

$$= Nf_j(t)\ (1-F_j(t))^{N-1}$$

$$= N\lambda_j\ \exp(-\lambda_j t)\ (\exp(-\lambda_j t))^{N-1}$$

$$= N\lambda_j\ \exp(-N\lambda_j t)$$

$$= \beta_j\ \exp(-\beta_j t),\ \text{where } \beta_j = N\lambda_j.$$

$$\text{(A1.5)}\quad R_j^{(1)}(t) = \int_t^\infty f_j^{(1)}(t)\ dt$$

$$= \exp(-\beta_j t).$$

---

[1]The expression for $f_j^{(i)}(t)$, when $i > N$ and $f_j(t)$ is not an exponential density function, is an approximation. It is based on the fact that the maximum sample size is $N+m_j$, even though no more than N components are operating at any time. For the exponential it is exact.

APPENDIX II

TABLES OF THE NORMALIZED PRESENT WORTH OF THE REPAIR/REPLACEMENT COSTS

TABLE A2.1 Normalized Present Worth of Repair/Replacement Costs with MTTR=0.10.

MTBF	MTTR
1.00	.10

TIME / IR	1	2	3	4	5	6	7	8	9	10
10	1.09	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14
11	1.09	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14
12	1.08	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13
13	1.08	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13	1.13
14	1.07	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
15	1.07	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11
16	1.06	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11
17	1.06	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10
18	1.05	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10
19	1.05	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09
20	1.04	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09

TABLE A2.2 Normalized Present Worth of Repair/Replacement Costs with MTTR=0.20.

MTBF	MTTR
1.00	.20

TIME / IR	1	2	3	4	5	6	7	8	9	10
10	1.26	2.08	2.17	2.17	2.17	2.17	2.17	2.17	2.17	2.17
11	1.25	2.06	2.15	2.15	2.15	2.15	2.15	2.15	2.15	2.15
12	1.24	2.05	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.13
13	1.24	2.03	2.11	2.11	2.11	2.11	2.11	2.11	2.11	2.11
14	1.23	2.01	2.09	2.09	2.09	2.09	2.09	2.09	2.09	2.09
15	1.22	1.99	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07
16	1.21	1.97	2.05	2.05	2.05	2.05	2.05	2.05	2.05	2.05
17	1.21	1.96	2.03	2.03	2.03	2.03	2.03	2.03	2.03	2.03
18	1.20	1.94	2.01	2.01	2.01	2.01	2.01	2.01	2.01	2.01
19	1.19	1.92	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
20	1.18	1.90	1.97	1.98	1.98	1.98	1.98	1.98	1.98	1.98

TABLE A2.3 Normalized Present Worth of Repair/Replacement Costs with MTTR=0.25.

MTBF	MTTR
1.00	.25

TIME / IR	1	2	3	4	5	6	7	8	9	10
10	1.14	2.34	2.62	2.65	2.65	2.65	2.65	2.65	2.65	2.65
11	1.13	2.32	2.59	2.62	2.62	2.62	2.62	2.62	2.62	2.62
12	1.12	2.30	2.56	2.59	2.59	2.59	2.59	2.59	2.59	2.59
13	1.11	2.27	2.53	2.56	2.56	2.56	2.56	2.56	2.56	2.56
14	1.11	2.25	2.50	2.53	2.53	2.53	2.53	2.53	2.53	2.53
15	1.10	2.23	2.47	2.50	2.50	2.50	2.50	2.50	2.50	2.50
16	1.09	2.20	2.44	2.47	2.47	2.47	2.47	2.47	2.47	2.47
17	1.09	2.18	2.41	2.44	2.44	2.44	2.44	2.44	2.44	2.44
18	1.08	2.16	2.39	2.41	2.41	2.41	2.41	2.41	2.41	2.41
19	1.07	2.14	2.36	2.38	2.38	2.38	2.38	2.38	2.38	2.38
20	1.06	2.11	2.33	2.35	2.36	2.36	2.36	2.36	2.36	2.36

TABLE A2.4 Normalized Present Worth of Repair/Replacement Costs with MTTR=0.33.

MTBF	MTTR
1.00	.33

TIME IR	1	2	3	4	5	6	7	8	9	10
10	.92	2.45	3.17	3.36	3.39	3.40	3.40	3.40	3.40	3.40
11	.91	2.43	3.13	3.31	3.34	3.34	3.34	3.34	3.34	3.34
12	.90	2.40	3.08	3.26	3.29	3.29	3.29	3.29	3.29	3.29
13	.90	2.37	3.04	3.21	3.24	3.24	3.24	3.24	3.24	3.24
14	.89	2.34	3.00	3.16	3.19	3.19	3.19	3.19	3.19	3.19
15	.89	2.31	2.95	3.11	3.14	3.14	3.14	3.14	3.14	3.14
16	.88	2.29	2.91	3.06	3.09	3.09	3.09	3.09	3.09	3.09
17	.87	2.26	2.87	3.02	3.04	3.05	3.05	3.05	3.05	3.05
18	.87	2.24	2.83	2.97	3.00	3.00	3.00	3.00	3.00	3.00
19	.86	2.21	2.79	2.93	2.95	2.95	2.95	2.95	2.95	2.95
20	.86	2.19	2.75	2.88	2.91	2.91	2.91	2.91	2.91	2.91

APPENDIX III

CALCULATION OF STEADY STATE CANNIBALIZATION PROBABILITY

Consider the system illustrated in Figure 3.4 (p. 40), with $N = 2$, $P = 3$, $m_1 = 2$, $m_2 = 1$, and $m_3 = 1$.

(A3.1) $D_1(t) = \Pr\{t_1^{(3)} < t_2^{(2)}\} \Pr\{t_1^{(3)} < t_3^{(2)}\}$

(A3.2) $D_2(t) = \Pr\{t_2^{(2)} < t_1^{(3)}\} \Pr\{t_2^{(2)} < t_3^{(2)}\}$

(A3.3) $D_3(t) = \Pr\{t_3^{(2)} < t_1^{(1)}\} \Pr\{t_3^{(2)} < t_2^{(2)}\}$

(A3.4) $D_1(t) = \int_0^t \int_0^y f_1^{(3)}(x)\, f_2^{(2)}(y)\, dxdy \int_0^t \int_0^y f_1^{(3)}(x)\, f_3^{(2)}(y)\, dxdy$

(A3.5) $D_2(t) = \int_0^t \int_0^y f_2^{(2)}(x)\, f_1^{(3)}(y)\, dxdy \int_0^t \int_0^y f_2^{(2)}(x)\, f_3^{(2)}(y)\, dxdy$

(A3.6) $D_3(t) = \int_0^t \int_0^y f_3^{(2)}(x)\, f_1^{(3)}(y)\, dxdy \int_0^t \int_0^y f_3^{(2)}(x)\, f_2^{(2)}(y)\, dxdy$

(A3.7) $f_1^{(3)}(t) = 12\lambda_1(1 - e^{-\lambda_1 t})^2\, e^{-2\lambda_1 t}$

(A3.8) $f_2^{(2)}(t) = 2\lambda_2(1 - e^{-\lambda_2 t})\, e^{-1\lambda_2 t}$

$$\text{(A3.9)} \quad f_3^{(2)}(t) = 2\lambda_3(1 - e^{-\lambda_3 t}) \; e^{-1\lambda_3 t}$$

Substituting the correct forms of Equations (A3.7-A3.9) into Equations (A3.4-A3.6), and setting the upper limit of the integrations to infinity, yields the following:

$$\text{(A3.10)} \quad D_1(\infty) = \int_0^\infty \int_0^y 24\lambda_1\lambda_2(1-e^{-\lambda_1 x})^2 \; e^{-2\lambda_1 x} \; (1-e^{-\lambda_2 y}) \; e^{-1\lambda_2 y} \; dxdy$$

$$X \int_0^\infty \int_0^y 24\lambda_1\lambda_3(1-e^{-\lambda_1 x})^2 \; e^{-2\lambda_1 x} \; (1-e^{-\lambda_3 y}) \; e^{-1\lambda_3 y} \; dxdy$$

$$\text{(A3.11)} \quad D_2(\infty) = \int_0^\infty \int_0^y 24\lambda_1\lambda_2(1-e^{-\lambda_2 x}) \; e^{-1\lambda_2 x} \; (1-e^{-\lambda_1 y})^2 \; e^{-2\lambda_1 y} \; dxdy$$

$$X \int_0^\infty \int_0^y 4\lambda_2\lambda_3(1-e^{-\lambda_2 x}) \; e^{-1\lambda_2 x} \; (1-e^{-\lambda_3 y}) \; e^{-1\lambda_3 y} \; dxdy$$

$$\text{(A3.12)} \quad D_3(\infty) = \int_0^\infty \int_0^y 24\lambda_1\lambda_3(1-e^{-\lambda_3 x}) \; e^{-1\lambda_3 x} \; (1-e^{-\lambda_1 y})^2 \; e^{-2\lambda_1 y} \; dxdy$$

$$X \int_0^\infty \int_0^y 4\lambda_2\lambda_3(1-e^{-\lambda_3 x}) \; e^{-1\lambda_3} \; (1-e^{-\lambda_2 y}) \; e^{-1\lambda_2 y} \; dxdy$$

Performing the required double integrations, and multiplying, yields the steady state cannibalization probability as a function of the respective hazard rates, λ_1, λ_2, and λ_3.

(A3.13) $$D_1(\infty) = \{6\lambda_2(1/2\lambda_2 - 6/(2\lambda_1+1\lambda_2) + 6/(2\lambda_1+2\lambda_2) - 3/(4\lambda_1+1\lambda_2) + 3/(4\lambda_1+2\lambda_2) - 8/(3\lambda_1+2\lambda_2) + 8/(3\lambda_1+1\lambda_2))\} \\ \times\{2\lambda_3(1/2\lambda_3 - 6/(2\lambda_1+1\lambda_3) + 6/(2\lambda_1+2\lambda_3) - 3/(4\lambda_1+1\lambda_3) + 3/(4\lambda_1+2\lambda_3) - 8/(3\lambda_1+2\lambda_3) + 8/(3\lambda_1+1\lambda_3))\}$$

(A3.14) $$D_2(\infty) = \{12\lambda_1(1/12\lambda_1 - 2/(2\lambda_1+\lambda_2) + 1/(2\lambda_1+2\lambda_2) - 2/(3\lambda_1+2\lambda_2) + 2/(3\lambda_1+1\lambda_2) - 2/(4\lambda_1+1\lambda_2) + 1/(4\lambda_1+2\lambda_2))\} \\ \times\{2\lambda_3(1/2\lambda_3 - 2/(1\lambda_2+1\lambda_3) + 2/(1\lambda_2+2\lambda_3) - 1/(2\lambda_2+2\lambda_3) + 1/(2\lambda_2+1\lambda_3))\}$$

(A3.15) $$D_3(\infty) = \{12\lambda_1(1/12\lambda_1 - 2/(2\lambda_1+\lambda_3) + 1/(2\lambda_1+2\lambda_3) - 2/(3\lambda_1+2\lambda_3) + 2/(3\lambda_1+\lambda_3) - 2/(3\lambda_1+2\lambda_3) + 1/(4\lambda_1+2\lambda_3))\} \\ \times\{2\lambda_2(1/2\lambda_2 - 2/(\lambda_2+\lambda_3) + 2/(2\lambda_2+\lambda_3) - 1/(2\lambda_2+ 2\lambda_3) + 1/(\lambda_2+2\lambda_3))\}$$

APPENDIX IV

CALCULATION OF MARGINAL DENSITY FUNCTIONS OF TERMINAL FAILURE TIMES

In the example problem of Chapter V, the following probability density functions are implicitly used in the calculation of the cannibalization probability: $h_1^{(5)}(t)$, $h_1^{(6)}(t)$, $h_2^{(3)}(t)$, $h_2^{(4)}(t)$, $h_3^{(1)}(t)$, $h_3^{(2)}(t)$, and $h_3^{(3)}(t)$.

The term $h_1^{(5)}(t)$ is the probability density function of the time to the fifth failure of the first component type. Because the replacement time is non-zero, the required density function is the density of the sum of the sum of 5 ordered failure times, and 4 replacement times.

(A4.1) $$h_1^{(5)}(t) = f_1^{(1)}(t)^{*5} * g_1(t)^{*4},$$

where $f_1^{(1)}(t)^{*5}$ is the fifth multiple convolution of the probability density function of the first ordered failure time of the first component type; $g_1(t)^{*4}$ is the fourth multiple convolution of the first component's replacement density function.

The term $h_1^{(6)}(t)$ is defined as the probability density function of the time to the sixth failure of the first component type. It is calculated in a way similar to $h_1^{(5)}(t)$.

(A4.2) $$h_1^{(6)}(t) = f_1^{(1)}(t)^{*6} * g_1(t)^{*5}.$$

The term $h_2^{(3)}(t)$ is the probability density function of the time to the third failure of the second component type.

$$\text{(A4.3)} \quad h_2^{(3)}(t) = f_2^{(1)}(t)^{*3} * g_2(t)^{*2}.$$

The term $h_2^{(4)}(t)$ is the probability density function of the time to the fourth failure of the second component type.

$$\text{(A4.4)} \quad h_2^{(4)}(t) = f_2^{(1)}(t)^{*4} * g_2(t)^{*3}.$$

The term $h_3^{(1)}(t)$ is the probability density function of the time to the first failure of the third component type. It is calculated, using the method of order statistics, as follows:

$$\text{(A4.5)} \quad h_3^{(1)}(t) = \binom{2}{1} F_3(t)^0 \; f_3(t) \; (1-F_3(t))$$

$$= 2 \; f_3(t) \; (1-F_3(t))$$

$$= 2 \; \lambda_3 \exp(-\lambda_3 t) \; (1-(1-\exp(-\lambda_3 t)))$$

$$= 2 \; \lambda_3 \exp(-2\lambda_3 t).$$

The terms $h_3^{(2)}(t)$, and $h_3^{(3)}$, are the density functions of the second and third failures, respectively, of the third component type.

$$\text{(A4.6)} \quad h_3^{(2)}(t) = f_3^{(1)}(t)^{*2} * g_3(t).$$

$$\text{(A4.7)} \quad h_3^{(3)}(t) = f_3^{(1)}(t)^{*3} * g_3(t)^{*2}.$$

Under exponential failure and replacement density functions, the respective marginal density functions of the terminal failure times are calculated as follows; the calculations are based on the definition of the convolution operation given in Equation (3.5) (p. 26).

$$\text{(A4.8)} \quad h_1^{(5)}(t) = f_1^{(1)}(t)^{*5} * g_1(t)$$

$$= \beta_1 (\beta_1 t)^4 \frac{\exp(-\beta_1 t)}{4!} * \mu_1 (\mu_1 t)^3 \frac{\exp(-\mu_1 t)}{3!}$$

$$= \beta_1^5 \mu_1^4/(4!\ 3!) \left\{ 6t^4 \frac{\exp(-\beta_1 t)}{\alpha_1^4} \right.$$

$$- 106t^3 \frac{\exp(-\beta_1 t)}{\alpha_1^5} - 24t^3 \frac{\exp(-\mu_1 t)}{\alpha_1^5}$$

$$+ 720t^2 \frac{\exp(-\beta_1 t)}{\alpha_1^6} - 360t^2 \frac{\exp(-\mu_1 t)}{\alpha_1^6}$$

$$- 2880t \frac{\exp(-\beta_1 t)}{\alpha_1^7} - 2160t \frac{\exp(-\mu_1 t)}{\alpha_1^7}$$

$$\left. + 5040 \frac{\exp(-\beta_1 t)}{\alpha_1^8} - 5040 \frac{\exp(-\mu_1 t)}{\alpha_1^8} \right\},$$

where $\alpha_1 = \mu_1 - \beta_1$, and $\beta_1 = N\lambda_1$.

(A4.9) $$h_1^{(6)}(t) = f_1^{(1)}(t)^{*6} * g_1(t)^{*5}$$

$$= \beta_1(\beta_1 t)^5 \frac{\exp(-\beta_1 t)}{5!} * \mu_1(\mu_1 t)^4 \frac{\exp(-\mu_1 t)}{4!}$$

$$= \beta_1^6 \mu_1^5 /(5!\ 4!) \left\{ 24\ t^5 \frac{\exp(-\beta_1 t)}{\alpha_1^5} \right.$$

$$- 600\ t^4 \frac{\exp(-\beta_1 t)}{\alpha_1^6} + 120\ t^4 \frac{\exp(-\mu_1 t)}{\alpha_1^6}$$

$$+ 7200\ t^3 \frac{\exp(-\beta_1 t)}{\alpha_1^7} + 2880\ t^3 \frac{\exp(-\mu_1 t)}{\alpha_1^7}$$

$$- 50400\ t^2 \frac{\exp(-\beta_1 t)}{\alpha_1^8} + 30240\ t^2 \frac{\exp(-\mu_1 t)}{\alpha_1^8}$$

$$+ 201600\ t \frac{\exp(-\beta_1 t)}{\alpha_1^9} + 161280\ t \frac{\exp(-\mu_1 t)}{\alpha_1^9}$$

$$\left. - 362880 \frac{\exp(-\beta_1 t)}{\alpha_1^{10}} + 362880 \frac{\exp(-\mu_1 t)}{\alpha_1^{10}} \right\}$$

where $\alpha_1 = \mu_1 - \beta_1$, and $\beta_1 = N\lambda_1$.

(A4.10) $$h_2^{(3)}(t) = f_2^{(1)}(t)^{*3} * g_2(t)^{*2}$$

$$= \beta_2(\beta_2 t)^2 \frac{\exp(-\beta_2 t)}{2!} * (\mu_2^2 t) \exp(-\mu_2 t)$$

$$= \beta_2^3\ \mu_2^2/(2!) \left\{ t^2 \frac{\exp(-\beta_2 t)}{\alpha_2^2} - 4t \frac{\exp(-\beta_2 t)}{\alpha_2^2} \right.$$

$$-2t \frac{\exp(-\mu_2 t)}{\alpha_2^3} + 6 \frac{\exp(-\beta_2 t)}{\alpha_2^4}$$

$$\left. - \frac{6 \exp(-\mu_2 t)}{\alpha_2^4} \right\},$$

where $\alpha_2 = \mu_2 - \beta_2$, and $\beta_2 = N\lambda_2$.

(A4.11) $$h_2^{(4)}(t) = f_2^{(1)}(t)^{*4} * g_2(t)^{*3}$$

$$= \beta_2(\beta_2 t)^3 \frac{\exp(-\beta_2 t)}{3!} * \mu_2(\mu_2 t)^2 \frac{\exp(-\mu_2 t)}{2!}$$

$$= \beta_2^4 \mu_2^3/(3!\ 2!) \left\{2 t^3 \frac{\exp(-\beta_2 t)}{\alpha_2^3} \right.$$

$$- 18 t^2 \frac{\exp(-\beta_2 t)}{\alpha_2^4} + 6 t^2 \frac{\exp(-\mu_2 t)}{\alpha_2^4}$$

$$+ 72 t \frac{\exp(-\beta_2 t)}{\alpha_2^5} + 48 t \frac{\exp(-\mu_2 t)}{\alpha_2^5}$$

$$\left. - 120 \frac{\exp(-\beta_2 t)}{\alpha_2^6} + 120 \frac{\exp(-\mu_2 t)}{\alpha_2^6}\right\},$$

where $\alpha_2 = \mu_2 - \beta_2$, and $\beta_2 = N\lambda_2$.

(A4.12) $$h_3^{(2)}(t) = f_3^{(1)}(t)^{*2} * g_3(t)$$

$$= \beta_3(\beta_3 t) \exp(-\beta_3 t) * \mu_3 \exp(-\mu_3 t)$$

$$= \beta_3^2 \mu_3 \left\{\frac{t \exp(-\beta_3 t)}{\alpha_3} + \frac{\exp(-\mu_3 t) - \exp(-\beta_3 t)}{\alpha_3^2}\right\},$$

where $\alpha_3 = \mu_3 - \beta_3$, and $\beta_3 = N\lambda_3$.

APPENDIX V

NUMERICAL INTEGRATION USING GAUSSIAN QUADRATURE

A detailed analysis of Gaussian Quadrature may be found in Carnahan, et al. [8] . However, an outline follows, in which the details are illustrated for the double integrations necessary to calculate $D_1(t)$.

(A5.1) $D_1(t) = d_{12}(t)\ d_{13}(t)$,

where, assuming $(m_1, m_2, m_3) = (5,2,0)$,

(A5.2) $d_{12}(t) = \int_0^t \int_0^y h_1^{(6)}(x)\ h_2^{(3)}(y)\ dx\ dy$, and

$d_{13}(t) = \int_0^t \int_0^y h_1^{(6)}(x)\ h_3^{(1)}(y)\ dx\ dy$.

Essentially, Gauss-Legendre Quadrature involves converting integrals of the form $\int_a^b f(x)\ dx$, into the form $k \int_0^1 f(z)\ dz$, by a suitable change of variables; the latter integral is ultimately transformed into the form of a finite summation, $\sum_{i=1}^{M} U_i\ f(z_i)$. Specifically, let $z=(x-a)/(b-a)$, then $x=(b-a)z + a$, and $dx=(b-a)dz$; thus,

(A5.3) $\int_a^b f(x)\ dx = (b-a) \int_0^1 f((b-a)z+a)\ dz$.

Next, the integral, $\int_0^1 f((b-a)z+a)\ dz$ is transformed into a finite

summation,

$$\text{(A5.4)} \quad \int_0^1 f((b-a)z+a) = \sum_{i=1}^{M} U_i \, f((b-a)R_i + a),$$

where the R_i terms are the roots of the M^{th} order Legendre Polynomial on the interval $[0,1]$, and U_i is the corresponding weighting factor for the i^{th} root. The roots and weights of the 10^{th} order Legendre Polynomial are given below.

Index i	Root R_i	Weight U_i
1	0.0130467357	0.0333356721
2	0.0674683166	0.0747256745
3	0.1602952158	0.1095431812
4	0.2833023029	0.1346333596
5	0.4255628305	0.1477621132
6	0.5744371694	0.1477621132
7	0.7166976970	0.1346333596
8	0.8397047841	0.1095431812
9	0.9325316833	0.0747256745
10	0.9869532642	0.0333356721

To perform the double integration for $d_{12}(t)$, using Quadrature, one makes the following transformation: ($M=10$, for a 10^{th} order fit)

$$\text{(A5.5)} \quad d_{12}(t) = t \sum_{i=1}^{M} U_i \cdot (R_i t) \cdot h_2^{(3)}(R_i t) \sum_{j=1}^{M} U_j \cdot h_1^{(6)}(R_j \cdot (R_i t)).$$

APPENDIX VI

STATE TRANSITION ANALYSIS FOR CONFIGURATIONS WITH 4, 3, 2, 1, OR 0 SPARES

From Figure 5.2 (p. 104), the following differential equations may be constructed for a configuration with 4 spares.

$$
\begin{aligned}
&\text{(A6.1)} \quad \dot{S}_{2,4}(t) = -2\lambda S_{2,4}(t) \qquad\qquad S_{2,4}(0) = 1.0 \\
&\dot{S}_{1,4}(t) = -(\lambda+\mu)S_{1,4}(t) + 2\lambda S_{2,4}(t) \\
&\dot{S}_{0,4}(t) = -\mu S_{0,4}(t) + \lambda S_{1,4}(t) \\
&\dot{S}_{2,3}(t) = -2\lambda S_{2,3}(t) + \mu S_{1,4}(t) \\
&\dot{S}_{1,3}(t) = -(\lambda+\mu)S_{1,3}(t) + \mu S_{0,4}(t) + 2\lambda S_{2,3}(t) \\
&\dot{S}_{0,3}(t) = -\mu S_{0,3}(t) + \lambda S_{1,3}(t) \\
&\dot{S}_{2,2}(t) = -2\lambda S_{2,2}(t) + \mu S_{1,3}(t) \\
&\dot{S}_{1,2}(t) = -(\lambda+\mu)S_{1,2}(t) + \mu S_{0,3}(t) + 2\lambda S_{2,2}(t) \\
&\dot{S}_{0,2}(t) = -\mu S_{0,2}(t) + \lambda S_{1,2}(t) \\
&\dot{S}_{2,1}(t) = -2\lambda S_{2,1}(t) + \mu S_{1,2}(t) \\
&\dot{S}_{1,1}(t) = -(\lambda+\mu)S_{1,1}(t) + \mu S_{0,2}(t) + 2\lambda S_{2,1}(t) \\
&\dot{S}_{0,1}(t) = -\mu S_{0,1}(t) + \lambda S_{1,1}(t) \\
&\dot{S}_{2,0}(t) = -2\lambda S_{2,0}(t) + \mu S_{1,1}(t) \\
&\dot{S}_{1,0}(t) = -\lambda S_{1,0}(t) + \mu S_{0,1}(t) + 2\lambda S_{2,0}(t) \\
&\dot{S}_{0,0}(t) = -\mu S_{0,0}(t) + \lambda S_{1,0}(t) \\
&\dot{S}_{1,-1}(t) = \mu S_{0,0}(t).
\end{aligned}
$$

The time dependent probability of having 0, 1, or 2 components of the first type being up at any time is calculated as follows:

(A6.2) $q_{01} = S_{0,4}(t) + S_{0,3}(t) + S_{0,2}(t) + S_{0,1}(t) + S_{0,0}(t).$

(A6.3) $q_{11} = S_{1,4}(t) + S_{1,3}(t) + S_{1,2}(t) + S_{1,1}(t) + S_{1,0}(t) + S_{1,-1}(t).$

(A6.4) $q_{21} = S_{2,4}(t) + S_{2,3}(t) + S_{2,2}(t) + S_{2,1}(t) + S_{2,0}(t).$

From Figure 5.3 (p. 105), the following differential equations may be constructed for a configuration having 3 spares.

(A6.5)

$$\begin{aligned}
\dot{S}_{2,3}(t) &= -2\lambda S_{2,3}(t) \qquad\qquad S_{2,3}(0) = 1.0 \\
\dot{S}_{1,3}(t) &= -(\lambda+\mu)S_{1,3}(t) + 2\lambda S_{2,3}(t) \\
\dot{S}_{0,3}(t) &= -\mu S_{0,3}(t) + \lambda S_{1,3}(t) \\
\dot{S}_{2,2}(t) &= -2\lambda S_{2,2}(t) + \mu S_{1,3}(t) \\
\dot{S}_{1,2}(t) &= -(\lambda+\mu)S_{1,2}(t) + \mu S_{0,3}(t) + 2\lambda S_{2,2}(t) \\
\dot{S}_{0,2}(t) &= -\mu S_{0,2}(t) + \lambda S_{1,2}(t) \\
\dot{S}_{2,1}(t) &= -2\lambda S_{2,1}(t) + \mu S_{1,2}(t) \\
\dot{S}_{1,1}(t) &= -(\lambda+\mu)S_{1,1}(t) + \mu S_{0,2}(t) + 2\lambda S_{2,1}(t) \\
\dot{S}_{0,1}(t) &= -\mu S_{0,1}(t) + \lambda S_{1,1}(t) \\
\dot{S}_{2,0}(t) &= -2\lambda S_{2,0}(t) + \mu S_{1,1}(t) \\
\dot{S}_{1,0}(t) &= -\lambda S_{1,0}(t) + \mu S_{0,1}(t) + 2\lambda S_{2,0}(t) \\
\dot{S}_{0,0}(t) &= -\mu S_{0,0}(t) + \lambda S_{1,0}(t) \\
\dot{S}_{1,-1}(t) &= \mu S_{0,0}(t)
\end{aligned}$$

If component type 2 has a configuration with 3 spares, then its appropriate state differential equations are given in (A6.5), and the following equations describe the time dependent probability of 0, 1, or 2 components of the second type being up at any given time.

(A6.6) $q_{02} = S_{0,3}(t) + S_{0,2}(t) + S_{0,1}(t) + S_{0,0}(t).$

(A6.7) $q_{12} = S_{1,3}(t) + S_{1,2}(t) + S_{1,1}(t) + S_{1,0}(t) + S_{1,-1}(t).$

(A6.8) $q_{22} = S_{2,3}(t) + S_{2,2}(t) + S_{2,1}(t) + S_{2,0}(t).$

Figure 5.4 (p. 106) contains the state transition diagram for a configuration with 2 spares, which results in the following system of differential equations.

(A6.9)
$$
\begin{aligned}
\dot{S}_{2,2}(t) &= -2\lambda S_{2,2}(t) \qquad\qquad S_{2,2}(0) = 1.0 \\
\dot{S}_{1,2}(t) &= -(\lambda+\mu)S_{1,2}(t) + 2\lambda S_{2,2}(t) \\
\dot{S}_{0,2}(t) &= -\mu S_{0,2}(t) + \lambda S_{1,2}(t) \\
\dot{S}_{2,1}(t) &= -2\lambda S_{2,1}(t) + \mu S_{1,2}(t) \\
\dot{S}_{1,1}(t) &= -(\lambda+\mu)S_{1,1}(t) + \mu S_{0,2}(t) + 2\lambda S_{2,1}(t) \\
\dot{S}_{0,1}(t) &= -\mu S_{0,1}(t) + \lambda S_{1,1}(t) \\
\dot{S}_{2,0}(t) &= -2\lambda S_{2,0}(t) + \mu S_{1,1}(t) \\
\dot{S}_{1,0}(t) &= -\lambda S_{1,0}(t) + \mu S_{0,1}(t) + 2\lambda S_{2,0}(t) \\
\dot{S}_{0,0}(t) &= -\mu S_{0,0}(t) + \lambda S_{1,0}(t) \\
\dot{S}_{1,-1}(t) &= \mu S_{0,0}(t)
\end{aligned}
$$

If component type 2 has a configuration with 2 spares, then the following equations describe the time dependent probability of 0, 1, or 2 components of the second type being up at any given time.

(A6.10) $q_{02} = S_{0,2}(t) + S_{0,1}(t) + S_{0,0}(t).$

(A6.11) $q_{12} = S_{1,2}(t) + S_{1,1}(t) + S_{1,0}(t) + S_{1,-1}(t).$

(A6.12) $q_{22} = S_{2,2}(t) + S_{2,1}(t) + S_{2,0}(t).$

Similarly, when component type 3 assumes a configuration with 2 spares,

(A6.13) $q_{03} = S_{0,2}(t) + S_{0,1}(t) + S_{0,0}(t).$

(A6.14) $q_{13} = S_{1,2}(t) + S_{1,1}(t) + S_{1,0}(t) + S_{1,-1}(t)$

(A6.15) $q_{23} = S_{2,2}(t) + S_{2,1}(t) + S_{2,0}(t).$

(One need only use λ_3, and μ_3, in place of λ_2, and μ_2, in the solution of the $S_{i,k}(t)$ equations)

Figure 5.5 (p. 107) contains the state transition diagram for configurations having 1 spare and 0 spares initially provisioned. When component type 3 has 1 spare, the following differential equations apply.

$$\text{(A6.16)}\quad \begin{aligned} \dot{S}_{2,1}(t) &= -2\lambda S_{2,1}(t) \qquad\qquad S_{2,1}(0) = 1.0 \\ \dot{S}_{1,1}(t) &= -(\lambda+\mu)S_{1,1}(t) + 2\lambda S_{2,1}(t) \\ \dot{S}_{0,1}(t) &= -\mu S_{0,1}(t) + \lambda S_{1,1}(t) \\ \dot{S}_{2,0}(t) &= -2\lambda S_{2,0}(t) + \mu S_{1,1}(t) \\ \dot{S}_{1,0}(t) &= -\lambda S_{1,0}(t) + \mu S_{0,1}(t) + 2\lambda S_{2,0}(t) \\ \dot{S}_{0,0}(t) &= -\mu S_{0,0}(t) + \lambda S_{1,0}(t) \\ \dot{S}_{1,-1}(t) &= S_{0,0}(t) \end{aligned}$$

$$\text{(A6.17)}\quad q_{03} = S_{0,1}(t) + S_{0,0}(t)$$

$$\text{(A6.18)}\quad q_{13} = S_{1,1}(t) + S_{1,0}(t) + S_{1,-1}(t)$$

$$\text{(A6.19)}\quad q_{23} = S_{2,1}(t) + S_{2,0}(t)$$

With 0 spares, the following equations apply.

$$\text{(A6.20)}\quad \begin{aligned} \dot{S}_{2,0}(t) &= -2\lambda S_{2,0}(t) \qquad\qquad S_{2,0}(0) = 1.0 \\ \dot{S}_{1,0}(t) &= -\lambda S_{1,0}(t) + 2\lambda S_{2,0}(t) \\ \dot{S}_{0,0}(t) &= -\mu S_{0,0}(t) + \lambda S_{1,0}(t) \\ \dot{S}_{1,-1}(t) &= \mu S_{0,0}(t) \end{aligned}$$

$$\text{(A6.21)}\quad q_{03} = S_{0,0}(t)$$

$$\text{(A6.22)}\quad q_{13} = S_{1,0}(t) + S_{1,-1}(t)$$

$$\text{(A6.23)}\quad q_{23} = S_{2,0}(t)$$

APPENDIX VII

COMPUTER PROGRAM FOR EXAMPLE PROBLEM OF CHAPTER V

1. First, it reads MM, and NN, the respective orders of the integration, where MM is the order of the outer summations, and NN is the order of the inner summations. Format(I2,1X,I2).

2. Next, it reads MM cards each with the Roots and Weights of the Quadrature factors in the Format(F11.11,6X,F11.11).

3. Then, it reads P cards (P=3) each having the following parameters:
   a) MTBF(J), MTTR(J), in Format(F20.8,1X,F20.8).
   b) A(J), the procurement cost for component J, in Format(F10.2).
   c) B(J), the repair/replacement cost for component J, in Format(F10.2).
   d) C(J), the cannibalization cost for component J, in Format(F10.2).

4. Then, it reads the revenue values, V(1), and V(2), for having 1, and 2 units up, respectively, in Format(F10.2,1X,F10.2).

5. After this, it reads the sets of feasible spare allocations, M(1), M(2), and M(3), in Format(I1,1X,I1,1X,I1).

6. The output and program documentation are self-explanatory.

```
//SOLOMON JOB (Q910,RRA,S01,006,JS),'JPS'                                  JOB5958
//*PASSWORD      **************************************************
//*GENREPRO
//$DATA
      IMPLICIT REAL*8(A-H,O-Z)
      REAL KO,K1,K2,K3,K4
      INTEGER P
      DIMENSION XMTBF(10),XMTTR(10)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON/COST/ A(10),B(10),C(10),V(10)
      COMMON /FUNCT/ALPHA(10),BETA(10)
      COMMON/GAUSS/RII(12),RJJ(12),U(12),W(12),MM,NN
      COMMON/MAIN/ N,P
      READ(5,101) MM,NN
101   FORMAT(I2,1X,I2)
      DO 105 II=1,MM
      READ(5,110)  RII(II), U(II)
110   FORMAT(F11.11, 6X,F11.11)
105   CONTINUE
      DO  125  II=1,MM
      RJJ(II)=RII(II)
      W(II)=U(II)
      WRITE(6,123) RII(II),U(II),    II
123   FORMAT(2X, F12.10,7X,F12.10,2X,3HII=,I3)
125    CONTINUE
C     ****************************************************************
C     * RJJ AND RII ARE THE ROOTS OF THE POLYNOMIAL USED FOR THE        *
C     * GAUSSIAN INTEGRATION. W AND U ARE THE RESPECTIVE WEIGHTS OR     *
C     * MULTIPLYING FACTORS REQUIRED TO PERFORM THE SINGLE INTEGRATIONS *
C     * IN SUBROUTINES COST2, COST3,AND COST4, AS WELL AS FOR THE       *
C     * DOUBLE INTEGRATIONS PERFORMED IN FUNCTIONS D12,D13,D21,D23,D31, *
C     * AND D32.                                                        *
C     ****************************************************************
```

```
      P=3
      N=2
C     ***********************************************************************
C     * HERE THE RESPECTIVE MTBF AND MTTR VALUES ARE READ IN FOR EACH      *
C     * COMPONENT TYPE J. NEXT, THE COST COEFFICIENTS ARE READ IN.         *
C     * A(J) = PROCUREMENT COST OF THE J TH COMPONENT TYPE.                *
C     * B(J)= REPAIR/REPLACEMENT COST OF THE J TH COMPONENT TYPE.          *
C     * C(J) = CANNIBALIZATION COST OF THE J TH COMPONENT TYPE.            *
C     * V(1) IS THE RETURN OR REVENUE DERIVED, ON AN ANNUAL BASIS, FOR     *
C     * HAVING 1 UNIT UP, OR AVAILABLE AT TIME T.                          *
C     * V(2) IS THE RETURN OR REVENUE OBTAINED FROM HAVING 2 UNITS         *
C     * AVAILABLE AT TIME T.                                               *
C     * V(1) AND V(2) ARE USED IN SUBROUTINE COST4 TO OBTAIN THE NET       *
C     * PRESENT WORTH OF THE RETURN FUNCTION.                              *
C     ***********************************************************************
      DO 10 J=1,P
      READ(5,7) XMTBF(J),XMTTR(J)
7     FORMAT(F20.8,1X,F20.8)
      XL(J)=1.D00/XMTBF(J)
      XMU(J)=1.D00/XMTTR(J)
      BETA(J)=DFLOAT(N)*XL(J)
      ALPHA(J)=XMU(J)-BETA(J)
      READ(5,8) A(J)
8     FORMAT(F10.2)
      READ(5,9) B(J)
9     FORMAT(F10.2)
      READ(5,11) C(J)
11    FORMAT(F10.2)
10    CONTINUE
      READ(5,12) V(1),V(2)
12    FORMAT(F10.2,1X,F10.2)
      WRITE(6,13) V(1),V(2)
13    FORMAT(2X,5HV(1)=,F10.2,1X,5HV(2)=,1X,F10.2)
```

```
      DO 16 J=1,P
      WRITE(6,15) J,XMTBF(J),XL(J),XMTTR(J),XMU(J)
15    FORMAT(2X,I2,2X,F20.8,1X,F20.8,1X,F20.8,1X,F20.8)
      WRITE(6,17) A(J),B(J),C(J)
17    FORMAT(2X,2HA=,F10.2,1X,2HB=,F10.2,1X,2HC=,1X,F10.2)
16    CONTINUE
      DO 100 IR=10,20,5
      R=DFLOAT(IR)/100.D00
      DO 200 NT=5,15,5
      T=DFLOAT(NT)
      DO 20 I=1,4
      READ(5,14) M(1),M(2),M(3)
14    FORMAT(I1,1X,I1,1X,I1)
      WRITE(6,989)
989   FORMAT(1H1)
      WRITE(6,18) I,M(1),M(2),M(3),T,IR
18    FORMAT(2X,'FOR CASE NUMBER',I2,1X,
     1  'M(1)=',I2,1X,'M(2)=',I2,1X,'M(3)=',I2,2X,'TIME HORIZON=',F9.3,
     2 2X,'ANNUAL INTEREST RATE=',I3,' PERCENT')
      NM(1)=N+M(1)-1
      NM(2)=N+M(2)-1
      NM(3)=N+M(3)-1
      CALL COST1(K1,T)
      CALL COST2(K2,T)
      CALL COST3(K3,T)
      CALL COST4(K4,T)
      KO=K1+K2+K3-K4
      WRITE(6,995) M(1),M(2),M(3),T,IR,KO
      WRITE(6,996) XMTBF(1),XMTTR(1),XMTBF(2),XMTTR(2),XMTBF(3),XMTTR(3)
20    CONTINUE
200   CONTINUE
100   CONTINUE
```

```
995   FORMAT(2X,'FOR SPARE ALLOCATION ',2X,I2,1X,I2,1X,I2,1X,
     12X,'OVER A TIME HORIZON',F9.2,1X,'WITH INTEREST RATE (IR) =',I3,
     1    ' PERCENT',
     2  1X,/,26X,'PW(TOTAL COST)=KO=', G16.8)
996   FORMAT(/,2X,'THE PERFORMANCE PARAMETERS WERE',/,
     1 2X,'MTBF(1)=',F15.8,1X,'MTTR(1)=',F15.8,/,
     1 2X,'MTBF(2)=',F15.8,1X,'MTTR(2)=',F15.8,/,
     1 2X,'MTBF(3)=',F15.8,1X,'MTTR(3)=',F15.8)
      STOP
      END
      SUBROUTINE COST1(K1,T)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL K1
      INTEGER P
      COMMON  XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON/COST/A(10),B(10),C(10),V(10)
      COMMON /MAIN/N,P
C     ******************************************************************
C     * K1 IS THE PRESENT WORTH OF THE PROCUREMENT COST, WHICH IS THE   *
C     * TOTAL COST OF ALL THE COMPONENTS FOR THE N UNITS, AND MJ SPARES *
C     * FOR EACH COMPONENT TYPE J, J=1,P.  THERE IS NO DISCOUNTING,     *
C     * SINCE THE PROCUREMENT COST IS INCURRED AT TIME ZERO.            *
C     ******************************************************************
      K1=0.D00
      DO 100 J=1,P
      K1=K1 + A(J)*DFLOAT(M(J)) + DFLOAT(N)*A(J)
100   CONTINUE
      WRITE(6,200)  M(1),M(2),M(3),K1
200   FORMAT(/,       2X,20HFOR SPARE ALLOCATION, I2,1X,I2,1X,I2,1X,
     1  24HPW(PROCUREMENT COST)=K1=,1X,F13.2)
      RETURN
      END
```

```
      SUBROUTINE COST2(K2,T)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL K2
      INTEGER P
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON/COST/ A(10),B(10),C(10),V(10)
      COMMON/GAUSS/RII(12),RJJ(12),U(12),W(12),MM,NN
      COMMON/MAIN/ N,P
C     ******************************************************************
C     * K2 IS THE PRESENT WORTH OF THE EXPECTED REPAIR/REPLACEMENT      *
C     * COST. THERE CAN BE AS MANY AS (N+MJ-1) REPLACEMENTS FOR THE J TH*
C     * COMPONENT TYPE, IF CANNIBALIZATION IS PERMITTED.                *
C     * GAUSSIAN QUADRATURE IS USED WITH MM ROOTS, TO PERFORM THE       *
C     * PRESENT WORTH INTEGRATION OF THE REPAIR/REPLACEMENT COSTS. IR   *
C     * IS THE CONTINUOUS ANNUAL INTEREST RATE, IN 'PERCENT PER YEAR',  *
C     * R=IR/100                                                        *
C     ******************************************************************
      PWRC=0.D00
      TDUM=T
      DO 100 II=1,MM
      TII=RII(II)*TDUM
      RCST=0.D00
      DO 200 J=1,P
      NMM=NM(J)
      DO 300 I=1,NMM
      RCST=RCST+I*B(J)*RCONFG(J,I,TII)
300   CONTINUE
200   CONTINUE
      PWRC=PWRC+U(II)*RCST*DEXP(-R*TII)
100   CONTINUE
      PWRC=PWRC*T
      K2=PWRC
```

```
      WRITE(6,400) M(1),M(2),M(3),K2,T,IR
400   FORMAT(/,2X,20HFOR SPARE ALLOCATION, I2,1X,I2,1X,I2,1X,
     1   'PW(REPAIR/REPLACEMENT COST)=K2=',1X,F13.2,1X,'WITH T=',F9.2,
     2 1X,'AND IR=', I3,' PERCENT')
      RETURN
      END
      FUNCTION RCONFG(J,I,T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON /FUNCT/ALPHA(10),BETA(10)
C     ********************************************************************
C     * THIS FUNCTION CALCULATES THE MULTIPLE CONVOLUTIONS,              *
C     * ((FJ*GJ)**I)*RJ, WHICH ARE REQUIRED                              *
C     * FOR THE EXPECTED REPAIR / REPLACEMENT COST (K2) .                *
C     * FOR TYPE 1, WITH A MAXIMUM OF 5 SPARES, N+MJ-1=6, SO THERE CAN   *
C     * BE AS MANY AS 6 REPLACEMENTS (I=6) OF THE FIRST COMPONENT TYPE   *
C     ********************************************************************
      IF(I.EQ.1) RCONFG=H2(J,T)/BETA(J)
      IF(I.EQ.2) RCONFG=H3(J,T)/BETA(J)
      IF(I.EQ.3) RCONFG=H4(J,T)/BETA(J)
      IF(I.EQ.4) RCONFG=H5(J,T)/BETA(J)
      IF(I.EQ.5) RCONFG=H6(J,T)/BETA(J)
      IF(I.EQ.6) RCONFG=H7(J,T)/BETA(J)
      RETURN
      END
```

```
      SUBROUTINE COST3(K3,T)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL K3
      INTEGER P
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON/COST/ A(10),B(10),C(10),V(10)
      COMMON/GAUSS/RII(12),RJJ(12),U(12),W(12),MM,NN
      COMMON/MAIN/ N,P
C     ***********************************************************************
C     * THIS SUBROUTINE USES THE FUNCTION ROUTINES D1,D2, AND D3, TO       *
C     * CALCULATE THE NET PRESENT WORTH OF THE EXPECTED CANNIBALIZATION *
C     * COSTS, K3. THERE CAN BE AS MANY AS (N-1) CANNIBALIZATIONS FOR     *
C     * A SYSTEM OF N UNITS. GAUSSIAN QUADRATURE IS UTILIZED TO           *
C     * PERFORM THE PRESENT WORTH INTEGRATIONS.                            *
C     * IR IS THE CONTINUOUS ANNUAL INTEREST RATE, IN  'PER CENT          *
C     * PER YEAR'. R=IR/100.                                               *
C     ***********************************************************************
      D1PW1=0.D00
      D2PW2=0.D00
      D3PW3=0.D00
      TDUM=T
      DO 100 II=1,MM
      TII=RII(II)*TDUM
      D1PW1=D1PW1+U(II)*D1(TII)*DEXP(-R*TII)
      D2PW2=D2PW2+U(II)*D2(TII)*DEXP(-R*TII)
      D3PW3=D3PW3+U(II)*D3(TII)*DEXP(-R*TII)
      ANS1=D1(TII)
      ANS2=D2(TII)
      ANS3=D3(TII)
      ANSU=U(II)
      WRITE(6,90) II,TII, ANS1,  D1PW1,ANSU
90    FORMAT(/,2X,3HII=,I2,1X,4HTII=,F13.5,1X,3HD1=,E11.3,1X,
     1   6HD1PW1=,E11.3,2X,'U(II)=',F18.11)
```

```
      WRITE(6,91)       ANS2,D2PW2
      WRITE(6,92)       ANS3,D3PW3
91    FORMAT(26X,3HD2=,E11.3,1X,6HD2PW2=,E11.3)
92    FORMAT(26X,3HD3=,E11.3,1X,6HD3PW3=,E11.3)
100   CONTINUE
      D1PW1=TDUM*D1PW1
      D2PW2=TDUM*D2PW2
      D3PW3=TDUM*D3PW3
      XN1=DFLOAT(N-1)
      K3=XN1*(C(1)*D1PW1+C(2)*D2PW2+C(3)*D3PW3)
      WRITE(6,213) M(1),M(2),M(3),K3,T,IR
213   FORMAT(/,  2X,20HFOR SPARE ALLOCATION, I2,1X,I2,1X,I2,1X,
     1   'PW(CANNIBALIZATION COST)=K3=',G16.8,1X,'WITH T=',F9.2,1X,
     2   'AND IR=',I3,' PERCENT')
      RETURN
      END
      SUBROUTINE COST4(K4,T)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL K4
      DIMENSION S1(19),S2(19),S3(19)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON/COST/A(10),B(10),C(10),V(10)
      COMMON/GAUSS/RII(12),RJJ(12),U(12),W(12),MM,NN
C     *******************************************************************
C     * K4 IS THE EXPECTED RETURN OR REVENUE FUNCTION FOR HAVING I UNITS*
C     * OPERATIONAL AT TIME T. THIS MUST BE CUMULATED (INTEGRATED) OVER *
C     * THE TOTAL TIME HORIZON TO OBTAIN THE TOTAL NET PRESENT WORTH    *
C     * OF THE RETURN OR REVENUE FUNCTION.                              *
C     * GAUSSIAN QUADRATURE IS USED TO PERFORM THE INTEGRATION.         *
C     *******************************************************************
      TDUM=T
      R1PW1=0.D00
      R2PW2=0.D00
```

```
      DO 100 II=1,MM
      TII=RII(II)*TDUM
      CALL PROB(S1,S2,S3,PR1,PR2,TII)
      R1PW1=R1PW1+U(II)*PR1*DEXP(-R*TII)
      R2PW2=R2PW2+U(II)*PR2*DEXP(-R*TII)
100   CONTINUE
      R1PW1=R1PW1*T
      R2PW2=R2PW2*T
      PBAR=1.D00 - PR1 - PR2
      K4=V(1)*R1PW1+V(2)*R2PW2
      WRITE(6,213) M(1),M(2),M(3),K4,T,IR
213   FORMAT( / ,2X,20HFOR SPARE ALLOCATION, I2,1X,I2,1X,I2,2X,
     1   'PW(RETURN OR REVENUE FUNCTION)=K4=', G16.8,1X,'WITH T=',F9.2,
     2                        1X,'AND IR=',I3,' PERCENT')
      RETURN
      END
      FUNCTION D12(T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON/GAUSS/RII(12),RJJ(12),U(12),W(12),MM,NN
C     *********************************************************************
C     * THIS FUNCTION CALCULATES THE PROBABILITY THAT COMPONENT TYPE 1    *
C     * FAILED AND CAUSED A CANNIBALIZATION BEFORE COMPONENT TYPE 2       *
C     * FAILS.                                                            *
C     * IT USES GAUSSIAN QUADRATURE TO PERFORM THE DOUBLE INTEGRATION     *
C     * OF THE MARGINAL DENSITY FUNCTIONS, H1**5,AND H1**6, WITH H2**3,   *
C     * AND H2**4.                                                        *
C     *********************************************************************
      TMAX=T
      SUMOUT=0.D00
```

```
C     ************************************************************************
C     *M(1) IS EQUAL TO 5, OR 4                                              *
C     *M(2) WILL ASSUME THE VALUES    2, OR 3                                *
C     *M(2) IS ASSUMED EQUAL TO 3, INITIALLY                                 *
C     *H1**5 US EQUAL TO  H5(1,T)                                            *
C     *H1**6  IS EQUAL TO H6(1,T)                                            *
C     *H2**3 IS EQUAL TO H3(2,T)                                             *
C     *H2**4 IS EQUAL TO  H4(2,T)                                            *
C     ************************************************************************
      DO 300  II=1,MM
      TDUM=RII(II)*TMAX
      SUMIN=0.D00
      DO 400  JJ=1,NN
      YDUM=RJJ(JJ)*TDUM
      BI=YDUM
      IF(M(1) .EQ. 4) GO TO 390
      SUMIN=SUMIN+W(JJ)*H6(1,BI)
      GO TO 400
390   SUMIN=SUMIN+W(JJ)*H5(1,BI)
400   CONTINUE
      SUMIN=SUMIN*TDUM
      BO=TDUM
      IF(M(2).EQ. 2) GO TO 250
      SUMOUT=SUMOUT+U(II)*SUMIN*H4(2,BO)
      GO TO 290
250   SUMOUT=SUMOUT+U(II)*SUMIN*H3(2,BO)
290   CONTINUE
300   CONTINUE
      SUMOUT=SUMOUT*T
      D12=SUMOUT
      RETURN
      END
```

```
      FUNCTION D13(T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON/GAUSS/RII(12),RJJ(12),U(12),W(12),MM,NN
C     ******************************************************************
C     * THIS FUNCTION CALCULATES THE PROBABILITY THAT COMPONENT TYPE   *
C     * 1 FAILED AND CAUSED A CANNIBALIZATION BEFORE COMPONENT TYPE 3  *
C     * FAILS. IT USES GAUSSIAN QUADRATURE TO PERFORM THE DOUBLE       *
C     * INTEGRATIONS OF THE MARGINAL DENSITY FUNCTIONS, H1**5,         *
C     * AND H1**6, WITH H3**1, H3**2, AND H3**3.                       *
C     ******************************************************************
      TMAX=T
      SUMOUT=0.D00
C     ******************************************************************
C     *M(1) IS EQUAL TO 4, OR 5                                        *
C     *M(3) WILL ASSUME THE VALUES  0, 1, OR 2                         *
C     *M(3) IS ASSUMED EQUAL TO  0, INITIALLY                          *
C     *H1**6 IS EQUAL TO H6(1,T)                                       *
C     *H3**1 IS EQUAL TO H1(3,T)                                       *
C     *H3**2 IS EQUAL TO H2(3,T)                                       *
C     *H3**3 IS EQUAL TO H3(3,T)                                       *
C     ******************************************************************
      DO 301  II=1,MM
      TDUM=RII(II)*TMAX
      SUMIN=0.D00
      DO 401 JJ=1,NN
      YDUM=RJJ(JJ)*TDUM
      BI=YDUM
      IF(M(1) .EQ. 4) GO TO 391
      SUMIN=SUMIN+W(JJ)*H6(1,BI)
      GO TO 401
391   SUMIN=SUMIN+W(JJ)*H5(1,BI)
401   CONTINUE
```

```
      SUMIN=SUMIN*TDUM
      BO=TDUM
      IF(M(3) .EQ.1) GO TO 251
      IF(M(3).EQ.2) GO TO 261
      SUMOUT=SUMOUT+U(II)*SUMIN*H1(3,BO)
      GO TO 291
251   SUMOUT=SUMOUT+U(II)*SUMIN*H2(3,BO)
      GO TO 291
261   SUMOUT=SUMOUT+U(II)*SUMIN*H3(3,BO)
291   CONTINUE
301   CONTINUE
      SUMOUT=SUMOUT*T
      D13=SUMCUT
      RETURN
      END
      FUNCTION D1(T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
C     ******************************************************************
C     * D1 IS THE PROBABILITY THAT COMPONENT TYPE 1 CAUSED A           *
C     * CANNIBALIZATION.                                               *
C     ******************************************************************
      D1=D12(T)*D13(T)
      RETURN
      END
```

```
      FUNCTION D21(T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON/GAUSS/RII(12),RJJ(12),U(12),W(12),MM,NN
C     ***************************************************************
C     * THIS FUNCTION CALCULATES THE PROBABILITY THAT COMPONENT TYPE 2  *
C     * FAILS AND CAUSES A CANNIBALIZATION BEFORE COMPONENT TYPE 1 FAILS*
C     * IT USES GAUSSIAN QUADRATURE TO PERFORM THE DOUBLE INTEGRATIONS  *
C     * OF THE MARGINAL DENSITY FUNCTIONS H2**3, AND H2**4, WITH H1**5, *
C     * AND H1**6.                                                      *
C     ***************************************************************
      TMAX=T
      SUMOUT=0.D00
C     ***************************************************************
C     *M(1) IS EQUAL TO 4, OR 5                                        *
C     *M(2) WILL ASSUME THE VALUES    2, OR 3                          *
C     *M(2) IS ASSUMED EQUAL TO 3, INITIALLY                           *
C     * H1**5 IS EQUAL TO  H5(1,T)                                     *
C     *H1**6  IS EQUAL TO H6(1,T)                                      *
C     *H2**3 IS EQUAL TO H3(2,T)                                       *
C     *H2**4 IS EQUAL TO  H4(2,T)                                      *
C     ***************************************************************
      DO 300  II=1,MM
      TDUM=RII(II)*TMAX
      SUMIN=0.D00
      DO 400  JJ=1,NN
      YDUM=RJJ(JJ)*TDUM
      BI=YDUM
      IF(M(2) .EQ. 2) GO TO 350
      SUMIN=SUMIN+W(JJ)*H4(2,BI)
      GO TO 390
350   SUMIN=SUMIN+W(JJ)*H3(2,BI)
390   CONTINUE
400   CONTINUE
```

```
      SUMIN=SUMIN*TDUM
      BO=TDUM
      IF(M(1) .EQ. 4) GO TO 281
      SUMOUT=SUMOUT+U(II)*SUMIN*H6(1,BO)
281   SUMOUT=SUMOUT+U(II)*SUMIN*H5(1,BO)
290   CONTINUE
300   CONTINUE
      SUMOUT=SUMOUT*T
      D21=SUMOUT
      RETURN
      END
      FUNCTION D23(T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON/GAUSS/RII(12),RJJ(12),U(12),W(12),MM,NN
C     *****************************************************************
C     * THIS FUNCTION CALCULATES THE PROBABILITY THAT COMPONENT TYPE 2  *
C     * FAILS AND CAUSES A CANNIBALIZATION BEFORE COMPONENT TYPE 3 FAILS*
C     * IT USES GAUSSIAN QUADRATURE TO PERFORM THE DOUBLE INTEGRATIONS  *
C     * OF THE MARGINAL DENSITY FUNCTIONS H2**3, WITH H3**1, H3**2,     *
C     * AND H3**3, FOR ONE SPARE CONFIGURATION, AND THE INTEGRATIONS    *
C     * OF H2**4 WITH H3**1,H3**2,AND H3**3, FOR ANOTHER CONFIGURATION. *
C     *****************************************************************
       TMAX=T
       SUMOUT=0.D00
C     *****************************************************************
C     *M(2) WILL ASSUME THE VALUES    2, OR 3                          *
C     *M(2) IS ASSUMED EQUAL TO 3, INITIALLY                           *
C     *H2**3 IS EQUAL TO H3(2,T)                                       *
C     *H2**4 IS EQUAL TO  H4(2,T)                                      *
C     *****************************************************************
```

```
C     ********************************************************************
C     *M(3) WILL ASSUME THE VALUES  0, 1, OR 2                           *
C     *M(3) IS ASSUMED EQUAL TO  0, INITIALLY                            *
C     *H3**1 IS EQUAL TO H1(3,T)                                         *
C     *H3**2 IS EQUAL TO H2(3,T)                                         *
C     *H3**3 IS EQUAL TO H3(3,T)                                         *
C     ********************************************************************
      DO 300  II=1,MM
      TDUM=RII(II)*TMAX
      SUMIN=0.D00
      DO 400  JJ=1,NN
      YDUM=RJJ(JJ)*TDUM
      BI=YDUM
      IF(M(2) .EQ. 2) GO TO 350
      SUMIN=SUMIN+W(JJ)*H4(2,BI)
      GO TO 390
350   SUMIN=SUMIN+W(JJ)*H3(2,BI)
390   CONTINUE
400   CONTINUE
      SUMIN=SUMIN*TDUM
      BO=TDUM
      IF(M(3).EQ.1) GO TO 250
      IF(M(3) .EQ. 2) GO TO 260
      SUMOUT=SUMOUT+U(II)*SUMIN*H1(3,BO)
      GO TO 290
250   SUMOUT=SUMOUT+U(II)*SUMIN*H2(3,BO)
      GO TO 290
260   SUMOUT=SUMOUT+U(II)*SUMIN*H3(3,BO)
290   CONTINUE
300   CONTINUE
      SUMOUT=SUMOUT*T
      D23=SUMOUT
      RETURN
      END
```

```
      FUNCTION D2(T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R, M(10),NM(10),IR
C     ******************************************************************
C     * D2 IS THE PROBABILITY THAT COMPONENT TYPE 2 CAUSES A           *
C     * CANNIBALIZATION.                                               *
C     ******************************************************************
      D2=D21(T)*D23(T)
      RETURN
      END
      FUNCTION D31(T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON/GAUSS/RII(12),RJJ(12),U(12),W(12),MM,NN
C     ******************************************************************
C     * THIS FUNCTION CALCULATES THE PROBABILITY THAT COMPONENT TYPE 3 *
C     * FAILS AND CAUSES A CANNIBALIZATION BEFORE COMPONENT TYPE 1     *
C     * FAILS. IT USES GAUSIAN QUADRATURE TO PERFORM THE DOUBLE        *
C     * INTEGRATIONS OF THE MARGINAL DENSITY FUNCTIONS H3**1, H3**2,   *
C     * AND H3**3, WITH H1**5, AND H1**6.                              *
C     ******************************************************************
      TMAX=T
      SUMOUT=0.D00
C     ******************************************************************
C     *M(1) IS EQUAL TO 4, OR 5                                        *
C     *M(3) WILL ASSUME THE VALUES  0, 1, OR 2                         *
C     *M(3) IS ASSUMED EQUAL TO  0, INITIALLY                          *
C      H1**5 IS EQUAL TO  H5(1,T)                                      *
C     *H1**6 IS EQUAL TO H6(1,T)                                       *
C     *H3**1 IS EQUAL TO H1(3,T)                                       *
C     *H3**2 IS EQUAL TO H2(3,T)                                       *
C     *H3**3 IS EQUAL TO H3(3,T)                                       *
C     ******************************************************************
```

```
      DO 300 II=1,MM
      SUMIN=0.D00
      TDUM=RII(II)*TMAX
      DO 400  JJ=1,NN
      YDUM=RJJ(JJ)*TDUM
      BI=YDUM
      IF(M(3).EQ.1) GO TO 350
      IF(M(3) .EQ. 2) GO TO 360
      SUMIN=SUMIN+W(JJ)*H1(3,BI)
      GO TO 390
350   SUMIN=SUMIN+W(JJ)*H2(3,BI)
      GO TO 390
360   SUMIN=SUMIN+W(JJ)*H3(3,BI)
390   CONTINUE
400   CONTINUE
      SUMIN=SUMIN*TDUM
      BO=TDUM
      IF(M(1) .EQ. 4) GO TO 281
      SUMOUT=SUMOUT+U(II)*SUMIN*H6(1,BO)
      GO TO 290
281   SUMOUT=SUMOUT+U(II)*SUMIN*H5(1,BO)
290   CONTINUE
300   CONTINUE
      SUMOUT=SUMOUT*T
      D31=SUMOUT
      RETURN
      END
```

```
      FUNCTION D32(T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON/GAUSS/RII(12),RJJ(12),U(12),W(12),MM,NN
C     ******************************************************************
C     * THIS FUNCTION CALCULATES THE PROBABILITY THAT COMPONENT TYPE 3  *
C     * FAILS AND CAUSES A CANNIBALIZATION BEFORE COMPONENT TYPE 2 FAILS*
C     * IT USES GAUSSIAN QUADRATURE TO PERFORM THE DOUBLE INTEGRATIONS  *
C     * OF THE MARGINAL DENSITY FUNCTIONS, H3**1, H3**2, AND H3**3,     *
C     * WITH H2**3, AND H2**4.                                          *
C     ******************************************************************
      TMAX=T
      SUMOUT=0.D00
C     ******************************************************************
C     *M(3) WILL ASSUME THE VALUES  0, 1, OR 2                          *
C     *M(3) IS ASSUMED EQUAL TO  0, INITIALLY                           *
C     *H3**1 IS EQUAL TO H1(3,T)                                        *
C     *H3**2 IS EQUAL TO H2(3,T)                                        *
C     *H3**3 IS EQUAL TO H3(3,T)                                        *
C     ******************************************************************
C     ******************************************************************
C     *M(2) WILL ASSUME THE VALUES    2, OR 3                           *
C     *M(2) IS ASSUMED EQUAL TO 3, INITIALLY                            *
C     *H2**3 IS EQUAL TO H3(2,T)                                        *
C     *H2**4 IS EQUAL TO  H4(2,T)                                       *
C     ******************************************************************
      DO 300 II=1,MM
      TDUM=RII(II)*TMAX
      BO=TDUM
      SUMIN=0.
```

```
      DO 400  JJ=1,NN
      YDUM=RJJ(JJ)*TDUM
      BI=YDUM
      IF(M(3).EQ.1) GO TO 350
      IF(M(3) .EQ. 2) GO TO 360
      SUMIN=SUMIN+W(JJ)*H1(3,BI)
      GO TO 390
350   SUMIN=SUMIN+W(JJ)*H2(3,BI)
      GO TO 390
360   SUMIN=SUMIN+W(JJ)*H3(3,BI)
390   CONTINUE
400   CONTINUE
      SUMIN=SUMIN*TDUM
      BO=TDUM
      IF(M(2).EQ. 2) GO TO 250
      SUMOUT=SUMOUT+U(II)*SUMIN*H4(2,BO)
      GO TO 290
250   SUMOUT=SUMOUT+U(II)*SUMIN*H3(2,BO)
290   CONTINUE
300   CONTINUE
      SUMOUT=SUMOUT*T
      D32=SUMOUT
      RETURN
      END
      FUNCTION D3(T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
C     ***********************************************************************
C     * D3 IS THE PROBABILITY THAT COMPONENT TYPE 3 CAUSES A                *
C     * CANNIBALIZATION.                                                    *
C     ***********************************************************************
      D3=D31(T)*D32(T)
      RETURN
      END
```

```
      SUBROUTINE PROB(S1,S2,S3,PR1,PR2,T)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION S1(19),S2(19),S3(19)
      EXTERNAL ST1,ST2,ST3
      DIMENSION RI1(19),RI22(10),RI23(13),RI30(4),RI31(7),RI32(10)
      DIMENSION SM1(19),SM23(13),SM22(10),SM32(10),SM31(7),SM30(4)
      DIMENSION WK1(551),WK23(377),WK22(290),WK32(290),WK31(203)
      DIMENSION WK30(116)
      DIMENSION RI14(16),SM14(16),WK14(464)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
C     *****************************************************************
C     * THIS SUBROUTINE CALLS 'DREBS', AN 'IMSL' ROUTINE, WHICH USES    *
C     * THE METHOD OF BULLERISCH ANC STOVER TO SOLVE THE STATE          *
C     * TRANSITION EQUATIONS GIVEN IN SUBROUTINES ST1,ST2,AND ST3.      *
C     * THEY ARE WRITTEN AS A SYSTEM OF FIRST ORDER DIFFERENTIAL EQNS.  *
C     *****************************************************************
      HMINO=0.0001D00
      HMINI=0.0001D00
      EPSO=0.01D00
      EPSI=0.01D00
      JM=3
      BI=T
      HI=0.005D00
      IF(M(1) .EQ. 5) GO TO 1
      NEQ1=16
      DO 6 I=1,NEQ1
      S1(I)=0.D00
      SM14(I)=1.D00
    6 CONTINUE
      S1(16)=1.D00
      TI=0.D00
      JI=0
```

```
200   IF(HI .GT. BI-TI) HI=BI-TI
      CALL DREBS(ST1,S1,TI,NEQ1,JM,3,JI,HI,HMINI,EPSI,RI14,SM14,WK14,
     1  IER)
      IF(TI .LT. BI-HMINI) GO TO 200
      P01=S1(14)+S1(11)+S1(8)+S1(5)+S1(2)
      P11=S1(1)+S1(3)+S1(6)+S1(9)+S1(12)+S1(15)
      P21=S1(16)+S1(13)+S1(10)+S1(7)+S1(4)
1     NEQ1=19
      DO 5 I=1,NEQ1
      S1(I)=0.D00
      SM1(I)=1.D00
5     CONTINUE
      S1(19)=1.D00
      TI=0.D00
      JI=0
300   IF(HI.GT.BI-TI) HI=BI-TI
      CALL DREBS(ST1,S1,TI,NEQ1,JM,3,JI,HI,HMINI,EPSI,RI1,SM1,WK1,IER)
      IF(TI.LT.BI-HMINI) GO TO 300
      P01=S1(17)+S1(14)+S1(11)+S1(8)+S1(5)+S1(2)
      P11=S1(1)+S1(3)+S1(6)+S1(9)+S1(12)+S1(15)+S1(18)
      P21=S1(19)+S1(16)+S1(13)+S1(10)+S1(7)+S1(4)
      IF(M(2).EQ.3) GO TO 20
      DO 15 I=1,10
      S2(I)=0.D00
      SM22(I)=1.D00
15    CONTINUE
      S2(10)=1.D00
      HI=0.001D00
      HI=0.005D00
      TI=0.D00
      JI=0
      NEQ2=10
400   IF(HI.GT.BI-TI) HI=BI-TI
      CALL DREBS(ST2,S2,TI,NEQ2,JM,3,JI,HI,HMINI,EPSI,RI22,SM22,WK22,IER
     1   )
```

```
      IF(TI.LT.BI-HMINI) GO TO 400
      P02=S2(2)+S2(5)+S2(8)
      P12=S2(1)+S2(3)+S2(6)+S2(9)
      P22=S2(4)+S2(7)+S2(10)
      GO TO 30
20    DO 25 I=1,13
      S2(I)=0.D00
      SM23(I)=1.D00
25    CONTINUE
      S2(13)=1.D00
      NEQ2=13
      HI=0.001D00
      HI=0.005D00
      TI=0.D00
      JI=0
500   IF(HI.GT.BI-TI) HI=BI-TI
      CALL DREBS(ST2,S2,TI,NEQ2,JM,3,JI,HI,HMINI,EPSI,RI23,SM23,WK23,IER
     1    )
      IF(TI.LT.BI-HMINI) GO TO 500
      P02=S2(2)+S2(5)+S2(8)+S2(11)
      P12=S2(1)+S2(3)+S2(6)+S2(9)+S2(12)
      P22=S2(13)+S2(10)+S2(7)+S2(4)
30    CONTINUE
      IF(M(3)-1)  40,50,60
40    DO 45 I=1,4
      SM30(I)=1.D00
      S3(I)=0.D00
45    CONTINUE
      S3(4)=1.D00
      NEQ3=4
      HI=0.005D00
      TI=0.D00
      JI=0
```

```
600   IF(HI.GT.BI-TI) HI=BI-TI
      CALL DREBS(ST3,S3,TI,NEQ3,JM,3,JI,HI,HMINI,EPSI,RI30,SM30,WK30,IER
     1   )
      IF(TI.LT.BI-HMINI) GO TO 600
      P03=S3(2)
      P13=S3(1)+S3(3)
      P23=S3(4)
      GO TO 70
50    DO 55 I=1,7
      SM31(I)=1.D00
      S3(I)=0.D00
55    CONTINUE
      S3(7)=1.D00
      NEQ3=7
      HI=0.005D00
      TI=0.D00
      JI=0
700   IF(HI.GT.BI-TI) HI=BI-TI
      CALL DREBS(ST3,S3,TI,NEQ3,JM,3,JI,HI,HMINI,EPSI,RI31,SM31,WK31,IER
     1    )
      IF(TI.LT.BI-HMINI) GO TO 700
      P03=S3(2)+S3(5)
      P13=S3(1)+S3(3)+S3(6)
      P23=S3(4)+S3(7)
      GO TO 70
60    DO 65 I=1,10
      SM32(I)=1.D00
      S3(I)=0.D00
65    CONTINUE
      S3(10)=1.D00
      NEQ3=10
      HI=0.005D00
      TI=0.D00
      JI=0
```

```
800    IF(HI.GT.BI-TI) HI=BI-TI
       CALL DREBS(ST3,S3,TI,NEQ3,JM,3,JI,HI,HMINI,EPSI,RI32,SM32,WK32,IER
      1    )
       IF(TI.LT.BI-HMINI) GO TO 800
       P03=S3(2)       +S3(5)+S3(8)
       P13=S3(1)+S3(3)+S3(6)+S3(9)
       P23=S3(4)+S3(7)+S3(10)
70     PR2=P21*P22*P23
       PR0=P01*P02*P03+P01*P02*P13+P01*P12*P03
      1   +P01*P12*P13+P11*P02* P03
      2   +P11*P02*P13+P11*P12*P03 + P01*P02*P23
      3 +       P01*P22*P03+P01*P22*P23+P21*P02*P03
      4   +P21*P02*P23+P21*P22*P03+P01*P12*P23
      5    +P01*P22*P13 + P11*P02*P23+P11*P22*P03
      6   +P21*P02*P13+ P21*P12*P03
       PR1=1.D00-PR0-PR2
       RETURN
       END
       SUBROUTINE ST1(S1,T,NEQ,S1DOT)
       IMPLICIT REAL*8(A-H,O-Z)
       DIMENSION S1(19),S1DOT(19)
       COMMON XL(10),XMU(10)
       COMMON R,M(10),NM(10),IR
C      ******************************************************************
C      * COMPONENT TYPE 1 IS ASSUMED TO HAVE 5 SPARES INITIALLY, AND HAS *
C      * 19 POSSIBLE STATES, WITH STATE 1 INDICATING 1 COMPONENT UP AFTER*
C      * CANNIBALIZATION. STATES 4,7,10,13,16, & 19 ARE THE STATES       *
C      * HAVING 2 COMPONENTS UP AT TIME T, WITH S1(19)=1, AT TIME ZERO.  *
C      * STATES 1,3,6,9,12,15, & 18 ARE THE STATES HAVING 1 COMPONENT UP,*
C      * STATES 3,6,9,12,15, &18 ARE THE STATES HAVING 1 COMPONENT UP,   *
C      * AND STATES 2,5,8,11,14, & 17 ARE THE STATES HAVING ZERO         *
C      * COMPONENTS UP.                                                  *
C      ******************************************************************
```

```
      IF(M(1) .EQ. 4) GO TO 100
      S1DOT(1)=XMU(1)*S1(2)
      S1DOT(2)=-XMU(1)*S1(2)+XL(1)*S1(3)
      S1DOT(3)=-XL(1)*S1(3)+2.D00*XL(1)*S1(4)+XMU(1)*S1(5)
      S1DOT(4)=-2.D00*XL(1)*S1(4)+XMU(1)*S1(6)
      S1DOT(5)=-XMU(1)*S1(5)+XL(1)*S1(6)
      S1DOT(6)=-(XMU(1)+XL(1))*S1(6) + 2.D00*XL(1)*S1(7)+XMU(1)*S1(8)
      S1DOT(7)=-2.D00*XL(1)*S1(7) + XMU(1)*S1(9)
      S1DOT(8)=-XMU(1)*S1(8)+XL(1)*S1(9)
      S1DOT(9)=-(XMU(1)+XL(1))*S1(9)+2.D00*XL(1)*S1(10)+XMU(1)*S1(11)
      S1DOT(10)=-2.D00*XL(1)*S1(10)+XMU(1)*S1(12)
      S1DOT(11)=-XMU(1)*S1(11)+XL(1)*S1(12)
      S1DOT(12)=-(XMU(1)+XL(1))*S1(12)+2.D00*XL(1)*S1(13)+XMU(1)*S1(14)
      S1DOT(13)=-2.D00*XL(1)*S1(13) + XMU(1)*S1(15)
      S1DOT(14)=-XMU(1)*S1(14)+XL(1)*S1(15)
      S1DOT(15)=-(XMU(1)+XL(1))*S1(15)+2.D00*XL(1)*S1(16)+XMU(1)*S1(17)
      S1DOT(16)=-2.D00*XL(1)*S1(16)+XMU(1)*S1(18)
      S1DOT(17)=-XMU(1)*S1(17)+XL(1)*S1(18)
      S1DOT(18)=-(XMU(1)+XL(1))*S1(18)+2.D00*XL(1)*S1(19)
      S1DOT(19)=-2.D00*XL(1)*S1(19)
      GO TO 200
C     ******************************************************************
C     * IF THERE ARE ONLY 4 SPARES FOR COMPONENT TYPE 1, THEN THERE ARE *
C     * 16 POSSIBLE STATES, WITH STATE 1 INDICATING 1 COMPONENT UP AFTER*
C     * CANNIBALIZATION. STATES 4,7,10,13, & 16 ARE THE STATES HAVING 2 *
C     * COMPONENTS UP, WITH S1(16)=1, AT TIME 0.                        *
C     * STATES 1, 3, 6, 9, 12, & 15 HAVE 1 COMPONENT UP, WHILE STATES 2,*
C     * 5, 8, 11, & 14 HAVE ZERO COMPONENTS UP.                         *
C     ******************************************************************
```

```
100   S1DOT(1)=XMU(1)*S1(2)
      S1DOT(2)=-XMU(1)*S1(2)+XL(1)*S1(3)
      S1DOT(3)=-XL(1)*S1(3)+2.D00*XL(1)*S1(4)+XMU(1)*S1(5)
      S1DOT(4)=-2.D00*XL(1)*S1(4)+XMU(1)*S1(6)
      S1DOT(5)=-XMU(1)*S1(5)+XL(1)*S1(6)
      S1DOT(6)=-(XMU(1)+XL(1))*S1(6) + 2.D00*XL(1)*S1(7)+XMU(1)*S1(8)
      S1DOT(7)=-2.D00*XL(1)*S1(7) + XMU(1)*S1(9)
      S1DOT(8)=-XMU(1)*S1(3)+XL(1)*S1(9)
      S1DOT(9)=-(XMU(1)+XL(1))*S1(9)+2.D00*XL(1)*S1(10)+XMU(1)*S1(11)
      S1DOT(10)=-2.D00*XL(1)*S1(10)+XMU(1)*S1(12)
      S1DOT(11)=-XMU(1)*S1(11)+XL(1)*S1(12)
      S1DOT(12)=-(XMU(1)+XL(1))*S1(12)+2.D00*XL(1)*S1(13)+XMU(1)*S1(14)
      S1DOT(13)=-2.D00*XL(1)*S1(13) + XMU(1)*S1(15)
      S1DOT(14)=-XMU(1)*S1(14)+XL(1)*S1(15)
      S1DOT(15) = -(XMU(1)+XL(1))*S1(15) +2.D00*XL(1)*S1(16)
      S1DOT(16)=-2.D00*XL(1)*S1(16)
200   CONTINUE
      RETURN
      END
      SUBROUTINE ST2(S2,T,NEQ,S2DOT)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION S2(13),S2DOT(13)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
C     ****************************************************************
C     * COMPONENT TYPE 2 IS ASSUMED TO HAVE 3 SPARES INITIALLY, THUS   *
C     * THERE ARE 13 POSSIBLE STATES WITH STATE 1 INDICATING ONE       *
C     * COMPONENT UP AFTER CANNIBALIZATION.                            *
C     * STATES  4,7,10, &13 ARE THE STATES HAVING 2 COMPONENTS UP,     *
C     * WITH S2(13)=1, AT TIME 0. STATES 1,3,6,9,&12 HAVE ONLY 1 COM-  *
C     * PONENT UP, AND STATES  2,5,8, &11 HAVE ZERO COMPONENTS UP.     *
C     ****************************************************************
```

```
      IF(M(2).EQ.2) GO TO 100
      S2DOT(1)=XMU(2)*S2(2)
      S2DOT(2)=-XMU(2)*S2(2)+XL(2)*S2(3)
      S2DOT(3)=-XL(2)*S2(3)+2.D00*XL(2)*S2(4)+XMU(2)*S2(5)
      S2DOT(4)=-2.D00*XL(2)*S2(4)+XMU(2)*S2(6)
      S2DOT(5)=-XMU(2)*S2(5)+XL(2)*S2(6)
      S2DOT(6)=-(XMU(2)+XL(2))*S2(6)+2.D00*XL(2)*S2(7)+XMU(2)*S2(8)
      S2DOT(7)=-2.D00*XL(2)*S2(7)+XMU(2)*S2(9)
      S2DOT(8)=-XMU(2)*S2(8)+XL(2)*S2(9)
      S2DOT(9)=-(XMU(2)+XL(2))*S2(9)+2.D00*XL(2)*S2(10)+XMU(2)*S2(11)
      S2DOT(10)=-2.D00*XL(2)*S2(10)+XMU(2)*S2(12)
      S2DOT(11)=-XMU(2)*S2(11)+XL(2)*S2(12)
      S2DOT(12)=-(XMU(2)+XL(2))*S2(12)+2.D00*XL(2)*S2(13)
      S2DOT(13)=-2.D00*XL(2)*S2(13)
      GO TO 200
C     ***************************************************************
C     * IF THERE ARE ONLY 2 SPARES FOR COMPONENT TYPE 2, THEN THERE ARE *
C     * 10 POSSIBLE STATES, WITH STATE 1 INDICATING 1 COMPONENT UP AFTER*
C     * CANNIBALIZATION. STATES 4,7, &10 ARE THE STATES HAVING 2 COM-   *
C     * PONENTS UP, WITH S2(10)=1,AT TIME 0.                            *
C     * STATES 1,3,6, & 9 HAVE 1 COMPONENT UP, WHILE STATES 2,5, &8     *
C     * HAVE ZERO COMPONENTS UP.                                        *
C     ***************************************************************
```

```
100   S2DOT(1)=XMU(2)*S2(2)
      S2DOT(2)=-XMU(2)*S2(2)+XL(2)*S2(3)
      S2DOT(3)=-XL(2)*S2(3)+2.D00*XL(2)*S2(4)+XMU(2)*S2(5)
      S2DOT(4)=-2.D00*XL(2)*S2(4)+XMU(2)*S2(6)
      S2DOT(5)=-XMU(2)*S2(5)+XL(2)*S2(6)
      S2DOT(6)=-(XMU(2)+XL(2))*S2(6)+2.D00*XL(2)*S2(7)+XMU(2)*S2(8)
      S2DOT(7)=-2.D00*XL(2)*S2(7)+XMU(2)*S2(9)
      S2DOT(8)=-XMU(2)*S2(8)+XL(2)*S2(9)
      S2DOT(9)=-(XMU(2)+XL(2))*S2(9)+2.D00*XL(2)*S2(10)
      S2DOT(10)=-2.D00*XL(2)*S2(10)
200   CONTINUE
      RETURN
      END
      SUBROUTINE ST3(S3,T,NEQ,S3DOT)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION S3(10),S3DOT(10)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
C     ****************************************************************
C     * COMPONENT TYPE 3 IS ASSUMED TO HAVE 2 SPARES INITIALLY.        *
C     * THERE ARE 10 STATES POSSIBLE, WITH THE STATE NUMBER 1 BEING    *
C     * THE EXISTING STATE AFTER A CANNIBALIZATION WAS PERFORMED.      *
C     * STATES 4,7, &10 ARE THE STATES HAVING 2 COMPONENTS UP, WITH    *
C     * S3(10)=1, AT TIME 0. STATES 3,6, &9 ARE THE STATES WITH ONE    *
C     * COMPONENT UP.                                                  *
C     ****************************************************************
```

```
      IF(M(3).EQ.1) GO TO 100
      IF(M(3).EQ.0) GO TO 200
      S3DOT(1)=XMU(3)*S3(2)
      S3DOT(2)=-XMU(3)*S3(2)+XL(3)*S3(3)
      S3DOT(3)=-XL(3)*S3(3)+2.D00*XL(3)*S3(4)+XMU(3)*S3(5)
      S3DOT(4)=-2.D00*XL(3)*S3(4)+XMU(3)*S3(6)
      S3DOT(5)=-XMU(3)*S3(5)+XL(3)*S3(6)
      S3DOT(6)=-(XMU(3)+XL(3))*S3(6)+2.D00*XL(3)*S3(7)+XMU(3)*S3(8)
      S3DOT(7)=-2.D00*XL(3)*S3(7)+XMU(3)*S3(9)
      S3DOT(8)=-XMU(3)*S3(8)+XL(3)*S3(9)
      S3DOT(9)=-(XMU(3)+XL(3))*S3(9)+2.D00*XL(3)*S3(10)
      S3DOT(10)=-2.D00*XL(3)*S3(10)
      GO TO 300
C     ******************************************************************
C     * IF THERE IS ONLY 1 SPARE FOR COMPONENT 3, THEN THERE ARE 7     *
C     * POSSIBLE STATES, WITH S3(7)=1 AT TIME 0. STATES 4 & 7 HAVE 2   *
C     * COMPONENTS UP, STATES 1, 3, & 6 HAVE 1 COMPONENT UP, AND STATES *
C     * 2 & 5 HAVE ZERO COMPONENTS UP.                                  *
C     ******************************************************************
100   S3DOT(1)=XMU(3)*S3(2)
      S3DOT(2)=-XMU(3)*S3(2)+XL(3)*S3(3)
      S3DOT(3)=-XL(3)*S3(3)+2.D00*XL(3)*S3(4)+XMU(3)*S3(5)
      S3DOT(4)=-2.D00*XL(3)*S3(4)+XMU(3)*S3(6)
      S3DOT(5)=-XMU(3)*S3(5)+XL(3)*S3(6)
      S3DOT(6)=-(XMU(3)+XL(3))*S3(6) + 2.D00*XL(3)*S3(7)
      S3DOT(7)=-2.D00*XL(3)*S3(7)
```

```
      GO TO 300
C     ******************************************************************
C     * IF THERE ARE ZERO SPARES FOR COMPONENT 3 THEN THERE ARE ONLY    *
C     * 4 STATES, WITH S3(4)=1 AT TIME 0. STATE 4 HAS 2 COMPONENTS UP,  *
C     * STATES 1 & 3 HAVE 1 UP, AND STATE 2, ZERO UP.                   *
C     ******************************************************************
200   S3DOT(1)=XMU(3)*S3(2)
      S3DOT(2)=-XMU(3)*S3(2)+XL(3)*S3(3)
      S3DOT(3)=-XL(3)*S3(3)+2.D00*XL(3)*S3(4)
      S3DOT(4)=-2.D00*XL(3)*S3(4)
300   CONTINUE
      RETURN
      END
      FUNCTION H1(J,T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON /FUNCT/ALPHA(10),BETA(10)
      H1=BETA(J)*DEXP(-BETA(J)*T)
      RETURN
      END
      FUNCTION H2(J,T)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON /FUNCT/ALPHA(10),BETA(10)
      H2=((BETA(J)*XMU(J))/(BETA(J)-XMU(J)))*(BETA(J)*DEXP(-BETA(J)*T))*
     1      (TOEA(-ALPHA(J),T) - TOEA(0.,T))
      RETURN
      END
```

```
      FUNCTION H3(J,T)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL K3
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON /FUNCT/ALPHA(10),BETA(10)
      K3=((BETA(J)**3)*(XMU(J)**2)/2.D00)
      H3=(K3*DEXP(-XMU(J)*T))*(T*T2EA(ALPHA(J) ,T )-T3EA(ALPHA(J),T))
      RETURN
      END
      FUNCTION H4(J,T)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL K4
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON /FUNCT/ALPHA(10),BETA(10)
      K4=((BETA(J)**4)*(XMU(J)**3)/(3.D00*2.D00*2.D00))
      H4=(K4*DEXP(-XMU(J)*T))*
     1  ((T**2)*T3EA(ALPHA(J),T)
     2   - 2.D00*T*T4EA(ALPHA(J),T) + T5EA(ALPHA(J),T))
      RETURN
      END
      FUNCTION H5(J,T)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL K5
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON /FUNCT/ALPHA(10),BETA(10)
      K5=((BETA(J)**5)*(XMU(J)**4)/(4.D00*3.D00*2.D00*3.D00*2.D00 ))
      H5=(K5*DEXP(-XMU(J)*T))*
     1  (( T**3)*T4EA(ALPHA(J),T) -   3.D00*(T**2)*T5EA(ALPHA(J),T)
     2   + 3.D00*T*T6EA(ALPHA(J),T) - T7EA(ALPHA(J),T))
      RETURN
      END
```

```
      FUNCTION H6(J,T)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL K6
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON /FUNCT/ALPHA(10),BETA(10)
      K6=((BETA(J)**6)*(XMU(J)**5)/
     1  (5.D00*4.D00*3.D00*2.D00*4.D00*3.D00*2.D00   ))
      H6=(   DEXP(-XMU(J)*T)) *
     1 (K6*(T**4)*T5EA(ALPHA(J),T) - 4.D00*(T**3)*T6EA(ALPHA(J),T)*K6
     2  +6.D00*K6*(T**2)*T7EA(ALPHA(J),T) - 4.D00*K6*T*T8EA(ALPHA(J),T)+
     3      K6*T9EA(ALPHA(J),T))
      RETURN
      END
      FUNCTION H7(J,T)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL K7
      COMMON XL(10),XMU(10)
      COMMON R,M(10),NM(10),IR
      COMMON /FUNCT/ALPHA(10),BETA(10)
      K7=((BETA(J)**7)*(XMU(J)**6)/
     1  (6.D00*5.D00*4.D00*     3.D00*2.D00*5.D00*4.D00*3.D00*2.D00) )
      H7=(   DEXP(-XMU(J)*T))*
     1  (K7*(T**5)*T6EA(ALPHA(J),T) -
     2 5.D00*K7*(T**4)*T7EA(ALPHA(J),T) +
     3 10.D00*K7*(T**3)*T8EA(ALPHA(J),T)-10.D00*K7*(T**2)*T9EA(ALPHA(J),
     4  T) +5.D00*K7*T*  T10EA(ALPHA(J),T) - K7*T11EA(ALPHA(J),T))
      RETURN
      END
```

```
FUNCTION TOEA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
IF(A .NE. 0.D00) TOEA=(DEXP(A*T)-1.D00)/A
IF(A .EQ. 0.D00) TOEA = T
RETURN
END
FUNCTION T1EA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
T1EA=T*DEXP(A*T)/A - (DEXP(A*T)-1.D00)/(A*A)
RETURN
END
FUNCTION T2EA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
T2EA=(T**2)*DEXP(A*T)/A -(2.D00/A)*T1EA(A,T)
RETURN
END
FUNCTION T3EA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
T3EA=(T**3)*DEXP(A*T)/A -(3.D00/A)*T2EA(A,T)
RETURN
END
```

```
FUNCTION T4EA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
T4EA=(T**4)*DEXP(A*T)/A -4.D00*T3EA(A,T)/A
RETURN
END
FUNCTION T5EA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
T5EA=(T**5)*DEXP(A*T)/A -5.D00*T4EA(A,T)/A
RETURN
END
FUNCTION T6EA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
T6EA=(T**6)*DEXP(A*T)/A - 6.D00 *T5EA(A,T)/A
RETURN
END
FUNCTION T7EA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
T7EA=(T**7)*DEXP(A*T)/A -7.D00*T6EA(A,T)/A
RETURN
END
```

```
FUNCTION T8EA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
T8EA=(T**8)*DEXP(A*T)/A - 8.D00*T7EA(A,T)/A
RETURN
END
FUNCTION T9EA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
T9EA=(T**9)*DEXP(A*T)/A - 9.D00*T8EA(A,T)/A
RETURN
END
FUNCTION T10EA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
T10EA=(T**10)*DEXP(A*T)/A - 10.D00*T9EA(A,T)/A
RETURN
END
FUNCTION T11EA(A,T)
IMPLICIT REAL*8(A-H,O-Z)
COMMON XL(10),XMU(10)
COMMON R,M(10),NM(10),IR
T11EA=(T**11)*DEXP(A*T)/A - 11.D00*T10EA(A,T)/A
RETURN
END
```

AD-A038 110 ARMY ELECTRONICS COMMAND FORT MONMOUTH N J F/G 15/5
OPTIMAL SYSTEM SPARE CONFIGURATION BASED ON THE PRESENT WORTH O--ETC(U)
FEB 77 J P SOLOMOND
UNCLASSIFIED ECOM-4477 NL

3 OF 3
AD
A038110

END
DATE
FILMED
5-77

PROGRAM OUTPUT

```
FOR CASE NUMBER 1 M(1)= 4 M(2)= 2 M(3)= 1  TIME HORIZON=     5.000

ANNUAL INTEREST RATE= 10 PERCENT

FOR SPARE ALLOCATION 4  2  1 PW(PROCUREMENT COST)=K1=      124550.00

FOR SPARE ALLOCATION 4  2  1 PW(REPAIR/REPLACEMENT COST)=K2=        2451.68

WITH T=      5.00 AND IR= 10 PERCENT

II= 1 TII=        0.06523 D1=  0.329D-34 D1PW1=  0.109D-35  U(II)=     0.03333567210
                          D2=  0.235D-29 D2PW2=  0.779D-31
                          D3=  0.900D-30 D3PW3=  0.298D-31

II= 2 TII=        0.33734 D1=  0.220D-17 D1PW1=  0.159D-18  U(II)=     0.07472567450
                          D2=  0.374D-15 D2PW2=  0.270D-16
                          D3=  0.755D-17 D3PW3=  0.546D-18

II= 3 TII=        0.80148 D1=  0.395D-10 D1PW1=  0.400D-11  U(II)=     0.10954318120
                          D2=  0.491D-09 D2PW2=  0.496D-10
                          D3=  0.298D-11 D3PW3=  0.301D-12

II= 4 TII=        1.41651 D1=  0.183D-06 D1PW1=  0.214D-07  U(II)=     0.13463335960
                          D2=  0.631D-06 D2PW2=  0.738D-07
                          D3=  0.222D-08 D3PW3=  0.259D-09
```

```
II= 5 TII=      2.12781 D1=   0.156D-04 D1PW1=   0.189D-05   U(II)=      0.14776211320
                        D2=   0.284D-04 D2PW2=   0.346D-05
                        D3=   0.766D-07 D3PW3=   0.941D-08

II= 6 TII=      2.87219 D1=   0.167D-03 D1PW1=   0.204D-04   U(II)=      0.14776211320
                        D2=   0.218D-03 D2PW2=   0.276D-04
                        D3=   0.501D-06 D3PW3=   0.649D-07

II= 7 TII=      3.58349 D1=   0.606D-03 D1PW1=   0.774D-04   U(II)=      0.13463335960
                        D2=   0.660D-03 D2PW2=   0.897D-04
                        D3=   0.131D-05 D3PW3=   0.188D-06

II= 8 TII=      4.19852 D1=   0.124D-02 D1PW1=   0.167D-03   U(II)=      0.10954318120
                        D2=   0.122D-02 D2PW2=   0.178D-03
                        D3=   0.211D-05 D3PW3=   0.340D-06

II= 9 TII=      4.66266 D1=   0.185D-02 D1PW1=   0.254D-03   U(II)=      0.07472567450
                        D2=   0.173D-02 D2PW2=   0.259D-03
                        D3=   0.266D-05 D3PW3=   0.465D-06

II=10 TII=      4.93477 D1=   0.225D-02 D1PW1=   0.299D-03   U(II)=      0.03333567210
                        D2=   0.206D-02 D2PW2=   0.301D-03
                        D3=   0.293D-05 D3PW3=   0.525D-06
```

```
FOR SPARE ALLOCATION 4   2   1 PW(CANNIBALIZATION COST)=K3=    6.2393036

WITH T=      5.00 AND IR= 10 PERCENT

FOR SPARE ALLOCATION 4   2   1   PW(RETURN OR REVENUE FUNCTION)=K4=    46164.203

WITH T=      5.00 AND IR= 10 PERCENT

FOR SPARE ALLOCATION     4   2   1    OVER A TIME HORIZON      5.00

WITH INTEREST RATE (IR) = 10 PERCENT

                  PW(TOTAL COST)=K0=    80843.563


THE PERFORMANCE PARAMETERS WERE
MTBF(1)=       0.80128000 MTTR(1)=       0.10000000
MTBF(2)=       1.60200000 MTTR(2)=       0.12019000
MTBF(3)=      80.12800000 MTTR(3)=       0.12019000
```

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